ON

MAGNETS

BY

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Errata.

Page IV; line 6 from above, *for* at shorter distances *read* at these shorter distances.

—	V,	„	8 from below,	„	0,01279	*read*	0,01289.
—	VII	„	1 „ „	„	0,477mm	„	0,0477mm.
—	21	„	15 from above,	„	$N\iota\varkappa A$	„	$N\iota\varkappa E$.
—	27	„	6 from below,	„	9,70	„	0,70.
—	43	„	17 from above,	„	fire	„	five.
—	47	„	9 „ „	„	this mode	„	his mode.
—	55	„	2 from below,	„	0,55	„	0,53.
—	56	„	6 from above,	„	0,75	„	0,78.
—	68	„	10 from below,	„	606,8	„	706,8.
—	92	„	14 from above,	„	0,42	„	0,24.
—	105	„	7 „ „	„	B's 12,3	„	B's 13,2.
—	116	„	14 „ „	„	53,0	„	52,0.
—	„	„	16 „ „	„	3,54	„	3,36.
—	117	„	13 „ „	„	upon the periphery of the end-surface	*read*	upon the end-surface.

Some thirty years since I required, with a purely practical object in view, to know the relation in which the attraction of magnets and keepers is dependent on the intervening distance.

Though *Newton*, *Musschenbroek*, and other philosophers had previously undertaken measurements and computed formulæ for the supposed variation of the force of attraction with the distance, the fact was then wholly unknown to me, and I commenced a series of measurements for distances ranging from $^1/_{400}$ to $^2/_5$ of an inch, and found it could be expressed by the formula:

$$v = \frac{K}{d + \gamma + b\,(d + \gamma)^2}$$

in which v = amount of attraction;

K a constant that may be regarded as expressing the energy of attraction;

d the distance between the terminal surfaces of magnet and keeper, and

γ and b constants.

This formula holds good equally with bar and horseshoe-magnets, and with permanent as well as electro-magnets. Provided the three observations, from which the constants are deduced, be taken at distances sufficiently wide apart, the computed attraction will be found to agree very closely with that actually observed. The formula is however merely empirical, and in no way a rational expression for the law of attraction.

I did not proceed with any further investigation until recently.

Having fallen in with *Wiedemann's* treatise on galvanism and electro-magnetism, I happened to peruse the section embracing *Dub's* and *Tyndall's* experiments on the attraction of

magnets and armatures. Both these philosophers simply give the *results* of their measurements, without adding any formula for the relation between the distances and the attractions observed, and from these measurements I noticed that *Wiedemann* seemed to infer, that the figures in *Dub's* series denoted a relation *different* from that indicated by *Tyndall's*. By applying the above formula, however, both series were found to correspond, on the whole, very well, as will be seen by the following tables.

Dub's Observations with Electro-magnet 12" long 1" diam.

Distance between Polar Surfaces in inches	or thus.	Diameter of Armatures.							
		1 inch.				$\frac{3}{4}$ in.			
		Intensity = 36.		Intensity = 70.		Intensity = 36.		Intensity = 70.	
		Observed.	Computed.	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.
$^1/_{180}$	(1)	1.1	1.05	4.6	4.57	1.25	1.25	4.6	4.26
$^1/_{90}$	(2)	0.9	0.87	3.5	3.57	0.9	0.95	3.1	3.28
$^1/_{60}$	(3)	0.71	0.73	2.9	3.12	0.77	0.76	2.6	2.65
$^1/_{45}$	(4)	0.6	0.63	2.6	2.67	0.65	0.63	2.15	2.21
$^2/_{45}$	(8)	0.38	0.38	1.65	1.61	0.36	0.35	1.3	1.30
$^1/_{15}$	(12)	0.27	0.27	1.05	1.09	0.23	0.23	0.92	0.89
$^4/_{45}$	(16)	0.19	0.19			0.16	0.16		
$^1/_9$	(20)	0.15	0.15	0.6	0.61	0.12	0.12	0.52	0.52
$^2/_{15}$	(24)	0.11	0.12			0.10	0.10		
$^7/_{45}$	(28)	0.095	0.095			0.08	0.078		
$^8/_{45}$	(32)	0.08	0.079			0.06	0.065		
$^1/_5$	(36)	0.07	0.067	0.27	0.27	0.055	0.054	0.26	0.26
$^1/_3$	(60)			0.13	0.13			0.125	0.127
	Constants								
	γ	5.2		4.5		2.67		2.58	
	b	0.05154		0.04508		0.03868		0.01650	
	K	8.568		31.34		5.236		16.12	

In the column of Distances the numbers noted in brackets are those employed in calculating the constants.

Dub's Observations.

Distance between Polar Surfaces in inches	or thus.	Diameter of Armatures.							
		½ in.				⅜ in.			
		Intensity = 36.		Intensity = 70.		Intensity = 36.		Intensity = 70.	
		Obs.	Comp.	Obs.	Comp.	Obs.	Comp.	Obs.	Comp.
$^1/_{180}$	(1)	1.4	1.59	6.4	6.72	1.6	2.23	6.2	6.80
$^1/_{90}$	(2)	0.92	0.87	3.8	3.76	0.95	0.94	3.4	3.37
$^1/_{60}$	(3)	0.65	0.59	2.85	2.59	0.65	0.59	2.4	2.22
$^1/_{45}$	(4)	0.48	0.45	2	1.97	0.45	0.43	1.7	1.65
$^2/_{45}$	(8)	0.23	0.23	0.95	0.97	0.194	0.193	0.78	0.79
$^1/_{15}$	(12)	0.15	0.15	0.65	0.63	0.11	0.119	0.5	0.51
$^4/_{45}$	(16)	0.11	0.11			0.08	0.084		
$^1/_9$	(20)	0.084	0.085	0.36[1])	0.36	0.062	0.063	0.28	0.28
$^2/_{15}$	(24)	0.07	0.069			0.05	0.050		
$^7/_{45}$	(28)	0.062	0.058			0.044	0.041		
$^8/_{45}$	(32)	0.05	0.050			0.032	0.034		
$^1/_5$	(36)	0.04	0.043	0.174	0.172			0.136	0.139
$^1/_3$	(60)			0.085	0.087			0.073	0.071
		Constants							
	γ	0.21		0.30		— 0.25		0	
	b	0.00654		0.01154		0.01758		0.01181	
	K	1.941		8.870		1.688		7.031	

Tyndall's Observations.
with permanent magnet and spherical armature.

Distance between magn. and armature in $^1/_{1000}$ inch	Attraction.	
	Observed.	Computed.
2	150	153.5
5	75	73.9
10	40	39.5
15	27	26.8
20	20.25	20.28
25	16.25	16.27
30	13.50	13.55

Constants

$\gamma =$ 0.80
b 0.001059
K 430.9

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[1]) The Table gives 0.46; probably a misprint for 0.36.

From the above results it appears that, on the whole, the values computed differ but slightly from those observed. This, however, applies less to the smallest intervals in *Dub's* series, in which a trifling error in the determination of the distances makes a great difference in the computed attractions. And the attractions at *shorter* distances show, too, when compared together, a decided want of regularity in the rate of progression; this has even caused the constant γ — which from its nature must be a positive magnitude — in the 7th series to become negative, and in the 8th series zero. Probably Professor *Dub's* object was rather to arrive at the general relation respecting the variation of the attractive force with the distance and with the diameters of the armatures, than to attain great accuracy in the several observations.

In *Tyndall's* series the computed, tally more closely with the observed values; the greatest discrepancy – which, here too, appears in the observations at the shortest distances — amounts to $\pm 1\frac{1}{2}$ per cent.

When performed with sufficient care, there is, as previously stated, a very close agreement between the attractions observed and those computed by the above formula, even when the distances, as in *Dub's* series, are comparatively great. Here is tabulated a series of the results of such observations, performed with a bar-magnet and armature, whose length is 146^{mm}, diameter 10.5^{mm}; the unit of distance being 0.0477^{mm}, and the unit of force, the weight 1.541 grm.

Distance between polar surf.	Attraction = v.		Δ v.
	Observed.	Computed.	
0.1	42.70	42.50	— 0.20
0.6	39.70	39.91	+ 0.21
1.1	37.40	37.60	+ 0.20
1.6	35.40	35.52	+ 0.12
2.1	33.55	33.63	+ 0.08
2.6	31.95	32 00	+ 0.05
3.6	29.10	28.94	— 0.16
4.6	26.70	26.43	— 0.27
.5.6	24.35	24.28	— 0.07
6.6	22.40	22.40	0.00
7.6	20.75	20.80	+ 0.05
9.6	18.05	18.09	+ 0.04
11.6	16.00	15.96	— 0.04
14.6	13.62	13.47	— 0.15
17.6	11.56	11.57	+ 0.01
20.6	10.02	10.08	+ 0.06
23.6	8.90	8.88	— 0.02
27.6	7.60	7.62	+ 0.02
31.6	6.59	6.62	+ 0.03
36.6	5.62	5.65	+ 0.03
41.6	4.85	4.88	+ 0.03
47.6	4.10	4.16	+ 0.06
57.6	3.25	3.29	+ 0.04
67.6	2.63	2.67	+ 0.04
82.6	2.02	2.03	+ 0.01
97.6	1.60	1.60	— 0.00
117.6	1.23	1 22	— 0.01
147.6	0.90	0.86	— 0.04

Constants

$\gamma =$ 8.42

K 401.8

b 0.01279

Now it appeared to me that, by means of calculations based on accurate observations of the force of attraction between magnet and armature at different distances, it might be possible, partially at least, to ascertain the nature of the molecular disturbance in the magnet when subjected to the action of an armature, and again, when removed beyond its influence. And if, by means of some hypothetical assumption, close agreement could be esta-

blished between computed and observed attractions, and likewise between certain changes in the conditions under which the observations were performed and the corresponding values of the constants which enter into the formulæ and are dependent on those conditions, it would go far to support the probability of the hypothesis, or, at least, provide a basis for further investigations with a prospect of attaining the desired result.

Assuming either with *Weber*, that magnetic action is produced by the deflection of the polaric molecules, or, with *Ampère*, that it is produced by the deflection of the planes of rotation of the electric currents by which the molecules are assumed to be surrounded, I have set forth a number of suppositions, evidently possible, to account for the reciprocal action of magnets and armatures, and have computed the force of attraction according to each of these suppositions, from three or more points in an accurate series of observations. And in several cases there is close agreement between the observed and computed attractions; throughout, however, there are discrepancies, which, though trifling as to magnitude, are of a nature plainly proving them to be systematic and not accidental.

I will give one or two instances:

A

By formula $v = \dfrac{K^2}{d + \gamma \left(1 - \dfrac{\beta}{d + \gamma + b}\right)^2}$

for the attraction of two permanent magnets,

Distance $= d$	Attraction $= v$. Observed.	Computed.	Δ v
1.1	73.50	73.89	+ 0.39
1.6	71.05	70.99	— 0.06
2.1	68.50	68.29	- 0.21
2.6	66.30	65.85	— 0.54
3.1	64.00	63.47	- 0.53
3.6	61.70	61.31	-- 0.39
4.6	57.50	57.29	— 0.21
5.6	53.50	53.74	+ 0.16
6.6	50.20	50.54	+ 0.34
7.6	47.25	47.67	+ 0.42
8.6	44.75	45.09	+ 0.34
10.6	40.30	40.61	+ 0.31
12.6	37.00	36.87	— 0.13
15.6	32.35	32,27	— 0.08
18.6	28.50	28.61	+ 0.11
21.6	25.75	25.61	— 0.14
24.6	23.30	23.13	— 0.17
28.6	20.50	20.42	— 0.08
32.6	18.47	18.22	— 0.25
37.6	16.10	16.01	— 0.09
42.6	14.44	14.22	— 0.22
48.6	12.69	12.51	— 0.18
58.6	10.43	10.35	-- 0.08
68.6	8.90	8 78	— 0.12
83.6	7.20	7.09	— 0.11
98,6	5.95	5.92	— 0.03
118.6	4.78	4.71	— 0.07
148.6	3.68	3.74	+ 0.06
161.6	3.36	3.41	+ 0.05

Constants

γ = 13.0
K 20.707
b 101.4
β 41.38

B

By formula $v = \dfrac{K}{d + \gamma \sqrt{a^2 + d + \gamma}}$

for the attraction between magnet and armature of soft iron

Distance $= d$	Attraction $= v$. Observed.	Computed.	Δ v
0.1	42.70	43.45	+ 0.75
0.6	39.70	40.69	+ 0.99
1.1	37.40	38.30	+ 0.90
1.6	35.40	36.04	+ 0.64
2.1	33.55	34.04	+ 0.49
2.6	31.95	32.23	+ 0.28
3.6	29.10	29.10	0.00
4.6	26.70	26.47	— 0.23
5.6	24.35	24.22	— 0.13
6.6	22.40	22.27	- 0.13
7.6	20.75	20.61	— 0.14
9.6	18.05	17.83	— 0.22
11.6	16.00	15.66	— 0.34
14.6	13.62	13.14	— 0.48
17.6	11.56	11.24	- 0.32
20.6	10.02	9.77	— 0,25
23.6	8.90	8.60	— 0.30
27.6	7.60	7.37	— 0.23
31.6	6.59	6.41	— 0.18
36.6	5.62	5.48	— 0.14
41.6	4.85	4.75	— 0.10
47.6	4.10	4.07	— 0.03
57.6	3.25	3.25	0.00
67.6	2.63	2.67	+ 0.04
82.6	2.02	2.08	+ 0.06
97.6	1.60	1.67	+ 0.07
117.6	1.23	1.31	+ 0.08
147.6	0.90	0.96	+ 0.06

Constants

γ = 8.42
K 1996
a^2 20.56

Length of magnet and armature = 146^{mm}, diam^r = 10.5^{mm}, Unit of distance = 0.477^{mm}, Unit of force = 1.541 grams.

It will be observed that Δv in the series A, is first positive, then negative, again positive, then negative and finally positive; that generally, too, the differences are greatest in the middle of the positive or negative parts, decreasing as they approach the points where the signs change. A similar observation may be made with regard to series B. The fact of the differences in both series mostly having reverse signs for the corresponding parts, shows that they cannot arise from a systematic defect in the method of measurement, but are due to a disagreement between the actual law and the laws assumed in the construction of the formulæ. That such must be the case was additionally confirmed by the fact that the variation of the magnetic intensity and of other conditions did not influence the values of the constants in the manner they should have done, had the hypothetic assumption been correct.

After much labour, I found it hopeless to attempt the solution of the problem in the way originally contemplated: the reciprocal action of magnets and armatures was, it appeared, far too intricate a phenomenon to admit of explanation save after a detailed, systematic, and separate investigation of both magnet and armature, especially as to the relations, firstly, of the molecular moments throughout the mass of each, — a subject on which much that remained to be learned, would first have to be ascertained, — and secondly, of the disturbance in those relations, wrought by the reaction depending upon the variation in the distance between the components of the magnetic couple, concerning which probably nothing whatever was known.

Though I shall not particularize the many abortive attempts to ascertain, in the manner above mentioned, what takes place in a magnet when acted upon by an external magnetic force, I nevertheless deem it desirable here to draw attention to a circumstance which led to a preliminary task, that had necessarily to be first undertaken, but which does not strictly come within the sphere of those magnetic relations that form the subject of the present paper.

In the first empirical formula there is a constant, γ, with which the distance between the polar surfaces has to be increased. This constant enters into all the subsequent hypothetical formulæ. I assumed that $d + \gamma$ was equal to the distance between a transverse section of the magnet and another of the armature, where the resultants of the free magnetism on the polar surfaces and sides might be supposed to lie, — or that γ was the amount by which the distance between these resultants exceeds that between the polar surfaces. If this assumption were correct, it is evident that, according as the distance between the magnet and armature increases, γ would also increase, and $(d + \gamma)$ gradually tend to become equal to the distance between the magnetic centres of gravity of the magnet and armature, hence it would not be a *constant* magnitude. But though γ is a most important element at short distances between magnet and armature, and must be determined with rigid exactitude, it rapidly loses this significance with the increase of distance, so that an error in γ — itself but a small magnitude — can scarcely exert any sensible influence at great distances; and it has therefore been treated in the formulæ as a true constant.

Were my assumption as to the nature of the quantity represented by γ correct — the area of the terminal surfaces of a cylinder being proportional to the square of the radius, but the circumference proportional to the radius itself — γ would be greater in armatures of smaller than in armatures of larger diameter, provided they were equal in length; but the very reverse was found to be the case; and the force of attraction at the side of the smaller armature not being perceptibly less intense than that at the side of the larger one, it was clear that the character of γ must be different from that assumed.

In the computation of the reciprocal attraction between magnet and armature, based on the force with which the molecules would attract each other on the assumed conditions, the distance between the molecules was taken as equal to that between the polar surfaces or rather equal to $d + \gamma$. But this was true only of the

molecules in immediate juxtaposition, and not of those at their sides. Without, however, inquiring further into the relation, the effect upon the obliquely situated molecules was assumed to be simply an increase of the force exerted on those in actual juxtaposition, and it did not strike me that two circular planes might be subject to a law of attraction wholly distinct from that affecting two points.

The said relation connected with the value of γ necessitated, however, an investigation of all the circumstances that might possibly serve to elucidate its true import, thus giving rise to the question, whether the law of attraction for two disks might possibly be different from that for two points. And this, on consideration, proved to be so.

Meanwhile not being able to discover that this law of attraction had been previously investigated, I was compelled to undertake the task myself. The result — which appeared as a separate paper in the transactions of the Christiania Society of Sciences for 1875, p. 260 — shows that the attraction between two circular planes is a function of their radii, no less than of their distance of separation; that at short distances the rate of variation in the attraction is exceedingly slow, so much so, that it does not become even inversely proportional to the distance in the 1st power until at the interval of nearly a radius, after which, however, it tends asymptotically to become inversely proportional the squares of the distances. On the other hand, the reciprocal attraction of two magnetic molecules is, for all distances, in inverse proportion to the squares of their distances. (*Hansteen; Gauss*).

The signification of γ was now obvious: it was a magnitude with which the distance between the polar planes had to be increased in order to diminish, in just proportion, the rapidity with which, at small distances, the force of attraction varies with these distances. Thus, in place of the simple law of attraction for two points, another and more complicated law was found to prevail for magnets with plane ends, into which the radius of the plane enters as a factor, or rather, as will subsequently

appear, there are no less than three several elements of attraction, each a function of the form and dimensions of the polar planes, on which the reciprocal action of magnet and armature depend.

Before proceeding to the investigations into the nature and effects of magnetic attraction, I will briefly notice the various assumptions and views that form the basis of my mode of procedure.

According to *Weber's* theory, the molecules in a magnetic body constitute in themselves a series of elementary magnets, which, if no free magnetism be present, are so situated one towards the other as to admit of the poles being mutually saturated. The molecular axes must be supposed to lie in every possible direction, although at and near the surface of the body they cannot but be parallel to the surface, perfect saturation depending apparently on such a condition. When an external — say a positive — magnetic force is acting upon a body, it attracts the negative molecular poles and repels the positive, thus effecting a deflexion of the mobile molecules.

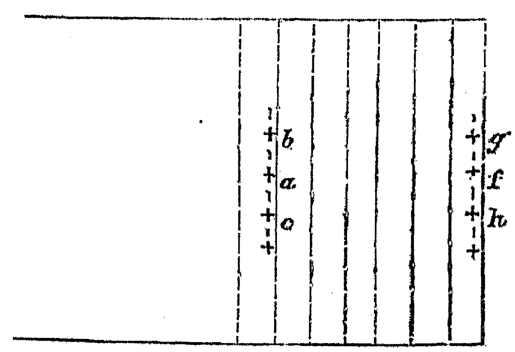
On withdrawing the external force, the molecules return to their original positions, provided the body be of soft iron; but if it consists of hard steel, this is only partially the case, and hence the molecules continue to occupy a deflected position in which they are no longer able to saturate one another completely, and in which their negative poles, to a greater extent than before, are directed *towards* the point whence the disturbing agency proceeded, the positive poles, on the other hand being directed *from* that point.

If the body acted upon be a bar of steel, towards one *end* of which the external deflecting force is directed, the position of the molecular axes becomes such, that their negative poles turn *towards*, and their positive poles *from* that end.

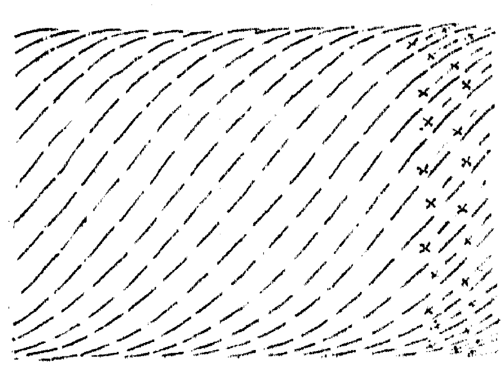
The interjacent molecules, may certainly be regarded as saturating one another in such a position also, but this cannot be the case with the molecules in the terminal strata. True, those in the terminal stratum, adjacent to the disturbeing body, are saturated,

either wholly or partially, by that body so long as it is there, but if removed, and the molecules are unable to regain their original positions, one of their poles in this stratum must necessarily become wholly or partially unsaturated, the degree of the non-saturation depending on the deflection the axes have experienced in the direction of the longitudinal axis of the bar.

A mode of grouping the molecules may be conceived, which, though it does not actually exist in nature, would produce an effect similar to that indicated. The molecules in the unmagnetized bar may be assumed to lie parallel to one another and at right angles to the axis of the bar. In a transverse section of a cylindrical bar the molecules would form a series of concentric rings. The positive pole of a molecule *a* would be saturated chiefly by the negative pole of the molecule *b*, but also by the negative poles of all the other molecules; its negative pole, by the positive pole of the molecule *c*, but also by the positive poles of all the other molecules. In the terminal stratum both the poles of a molecule, *f*, are in the same manner saturated from the left side, but the saturation is mainly effected by the adjacent apposite poles of the molecules *g* and *h*, and so on.



When acted upon by an external — say a positive — magnetic force, the molecules in such a body are deflected as shown in the accompanying diagram. With regard to the inner strata, the reciprocal saturation proceeds in the manner described above; but for the terminal strata the relation is a different one. For it is obvious, from the diagram, that the positive molecular poles in this stratum are more completely saturated than the negative poles, and hence there must be a surplus of free negative magnetism in the molecules throughout the whole of



this end-surface of the bar; and from the same cause, there must also be a surplus of positive magnetism left in the opposite end, the amount at each end being proportional to the *sine* of the molecular angle of deflection.

Doubtless there is no such grouping of the molecules as above supposed, but the effect of the relation thus indicated is precisely the same as when the molecular axes are situated in all possible directions. And as the treatment of the question will be greatly facilitated by adopting the above assumption, the position of the molecular axes, before subjection to magnetic influence, will in the folloving pages be assumed to be transversal to the longitudinal axes of the magnet and armature, and after induction, or magnetic disturbance, to be deflected from that position to an equal extent, throughout the whole of the same stratum at equal distances from the surface.

Hence the arm of the lever on which the magnetism of the molecule acts is proportional to the *sine* of the angle of deflection. And moreover, all the molecules being assumed to have their poles equally distant one from the other, the projection of the levers on the axis of the magnet will also be always represented by the *sine* of their angle of deflection.

Now free magnetism is, as is well known, proportional to the difference between the moments of the adjacent molecules or of the strata in which they lie.

Whatever is true of a single molecule applies equally to an annular element in a layer transverse to the longitudinal axis of a cylinder, such annular element constituting the sum of a series of molecules with the same angle of deflection. It will subsequently appear that the several annular elements in a layer are deflected at different angles, the deffection decreasing towards its centre. For any stratum however a mean angle of deflection may be assumed, whose *sine* is proportional to the magnetic moment of that stratum.

My calling in life being remote from that of a Philosopher, it is merely as a diversion or pastime, in leisure hours, that I can give my attention to subjects like the present. Having in consequence had but a limited time at my disposal, my investigations had necessarily to be extended over a period of several years. It could not, therefore, cause me any surprise to learn from time to time, that not a few of the questions that have formed parts of my examinations had, during that time, been treated by others, and the results published in scientific journals or separate papers. Thus, particularly, there have appeared in „*Comptes rendus*", articles, by *Jamin*, on investigations which, previous to their publication, had in part at least, been undertaken by myself. It appears nevertheless advisable here to add my own results, even though they be superseded by others in a more exhaustive form, because, being undertaken with a view to elucidate the special question which forms the principal object of the present treatise, they have naturally been conducted in a manner, considered most practical for this purpose.

During the progress of my pursuit I have frequently felt the want of special knowledge, for which it has been necessary to refer to others. In such cases I have applied to my friends, Professors Guldberg and Schjøtz, and feel a desire here to express my recognition of the constant readiness with which they, on all such occasions have met me, and for the liberality with which the latter gentleman has placed at my disposal the instrnments and a working room of the University, for making a great number of the necessary observations.

I.

As already stated in the introduction, the position of the molecules in a magnetic body may be regarded as if the mean angle of deflection in every stratum, perpendicular to any axis in the body, were equal to zero, so long as the body remains in an unmagnetised state. But when acted upon by an external magnetic force, the angle of deflection in all the strata perpendicular to the direction of the force acquires a definite value, the *sine* of that value indicating the magnetic moment of the stratum. On the magnetic force ceasing to act, a change results in the value of the angle of deflection, and two possibilities may be conceived: — 1st, the angle of deflection either again returns to zero, and the body thereby to its former unmagnetized state, or 2nd, the angle of deflection acquires a diminished value and the body continues magnetized. The latter arises when it is in possession of coercive force, which is the case in the highest degree with hardened steel, but also, though in a minor degree, with soft steel and hard iron. Even the softest iron is scarcely ever absolutely without coercive force, and does not therefore lose every trace of free magnetism, so soon as the magnetising force ceases to act.

Now, if a body possessing coercive power be exposed to external magnetic action, which changes the angles of deflection of the molecules, the angular change — as will appear from the experiments undertaken — is of two kinds, partly a *transient* and partly a *permanent* deflection. The *transient* change takes place within certain definite limits only, the extent of which is dependent

on the amount of deflection of the molecules, and ceases on the withdrawal of the force that occasioned it.

The *permanent* change is not effected till the above limits are exceeded, and its magnitude is therefore equal to the entire, minus the transient deflection. On the disturbing force being withdrawn, the permanent molecular deflection remains undiminished, provided its angle be not reduced by some other agency. But whether such reduction takes place or not, on the disturbing or deflecting force ceasing to act, the angle of deflection of the molecules is always diminished to an extent corresponding to the transient deflection. In soft iron, which has no coercive power, the molecular changes are always transient, the molecules immediately regaining their original neutral position, on the force which occasioned the deflection being withdrawn; but the change in soft iron must nevertheless be regarded as identical in nature with that, which in hard steel produces the permanent change; for both must be considered as the result of a deflection of the molecules, during which they *glide* or *rub* one against the other. But whereas the force, with which the molecules, after the magnetic action has ceased, seek to regain their original position, in order to neutralize the magnetic effect produced by one on the other, in soft iron is sufficient to effect, almost entirely, such restoration of the respective positions of the molecules, even when forming a closed circuit, or a system in which the two polar ends are connected with an armature, it is not sufficient to effect this restoration in the case of hard steel, because the coercive power or friction between the molecules offers a resistance sufficiently powerful to prevent it; and it is only in the event of the closed circuit being interrupted, i. e. on the ends of the magnet becoming free, whereby the force with which the molecules in the terminal surfaces endeavour to saturate each other is greatly increased, that this backward gliding of the molecules one upon the other can occur, a result, however, which even under such circumstances, does not always follow.

With the object of investigating these phenomena, but chiefly

the nature of the molecular disturbance in hard steel, the following experiments have been instituted.

A hardened bar of steel, 146^{mm} long and 10.5^{mm} in diameter, to the middle of which an induction coil had been attached, was placed between the polar ends of the large electro-magnet of the University, and the current from a Bunsen's cell, connected with the electro-magnet, having been closed, the effect of the inductive current produced by the instantaneous change in the position of the molecules of the bar, was read off on a mirror-galvanometer. The whole of the current was not, however, conducted through the galvanometer, the oscillations of the mirror would in that case have been too violent; the greater part was, therefore, got rid of by means of a shunt, leaving not more than $^1/_{13.9}$ of the whole current to pass through the galvanometer.

On the current from the cell being closed the deviation of the galvanometer was . . . . + 7.66

As soon as the mirror was at rest the current was again broken, the result being . . . . — 2.67

It was again closed, deviation . . . . + 2.63

again intercepted . . . . — 2.50

again closed. . . . . + 2.54

The steel bar, having been brought to a red heat and re-hardened, was again placed in the electro-magnet, which had, meanwhile, been connected with four cells, by which the magnetomotoric power was nearly doubled. On the current being closed, the deviation of the galvanometer was . . . . + 14.07

on its being broken . . . . — 2.27

— - closed . . . . + 2.29

— - broken . . . . — 2.20

— - closed . . . . + 2.29

After the current had been again broken and the magnetomotoric power increased to about $2^1/_2$ times that produced by one cell, on the current being closed, the result was . . . . + 3.20

on its being broken . . . . — 1.75

The steel bar was then moved lower down in the electro-magnet,

by which the magnetizing power of the latter was increased, and on the current being closed, the result was . . . . . . + 2.32
on its being broken . . . . — 1.80
— - closed . . . . + 1.84
— - broken, after the lapse of three hours. . . — 1.87
— - immediately closed. . . . . + 1.94

It will be seen from the first of these series, that after the molecules, at the moment the current was closed, had been deflected to an extent corresponding to the result 7.66 as shewn by the galvanometer, on the current being broken they retroceded through an angle indicated by 2.67. This proves the presence of a force, that rearranges the molecules when freed from the influence which caused their deflection. Now this force *might* be identical with that, which in soft iron causes the molecules to regain their original positions, when no longer acted upon by the deflecting energy. If so, the fact that the original neutral positions are not wholly regained in hardened steel, must be ascribed to the coercive power, or friction between the molecules, at the point where they stop, there counterbalancing the returning tendency, while the coercive force, at deviations *beyond* that point (the angle corresponding to 7.66—2.67), must not have proved sufficiently powerful in proportion, and consequently been overcome by the returning tendency, whose intensity is proportional to the degree of deflection. Were such actually the case, no repulsion could ensue on the interruption of the current, unless the angle of deflection, at which the returning tendency and the coercive power are in equilibrium, were exceeded; but that limit being passed, the retrogression would correspond to the whole arc, indicating such excess. Thus — supposing φ to be the deflection, and ψ the retrogression at the moment the current is interrupted, then, if the deflection were increased to $\varphi + \Delta\, \varphi$, the retrogression must be $\psi + \Delta\, \varphi$, as neither the coercive power, nor the angle, at which the retrogressive tendency and the coercive power are in equilibrium, can be dependent on the angle to which the molecules had been deflected. The experiments, however, give a very different result, and show that, at the three above degrees

of magnetization, the arcs of retrogression were not much different, although the total deflection by the last, was more then twice as great as by the first magnetization. So far are the deviations indicating the retrogression from being larger in proportion as the magnetomotoric energy and, therefore, the total deflection, have been increased, that quite the contrary is the case. This, however, does not imply that the arcs of the retrogression have a relation to each other, in any way inverse with the total deflections, for when those arcs are equal or even increasing to a certain degree, the deviations indicating them must necessarily decrease with the increase of the total deflection, as those deviations are proportional to the difference between the *sines* of the angle of molecular deflection before and after the interruption of the current in the electro-magnet.

It further appears from the above series of experiments, that the current being again closed, after having been interrupted, the deflection of the galvanometer was equal but in contrary direction to what it was, on the current being broken, whereas it was invariably greater the first time the current was opened, after the magnetizing force had been increased.

The assumption mentioned above, that the retrogressive movement of the molecules might be of the same nature as that which takes place in soft iron, seems not only inadequate to account for these phenomena, but the results of the experiments seem directly opposed to such an assumption; they seem plainly to show — that the force, which on the interruption of the current gives rise to the retrogression in hardened steel, is essentially different from that which in soft iron causes the molecules to regain their neutral positions; that a certain portion of the moment, imparted to the molecules of hardened steel by the magnetizing force, is *retained* in a closed magnetic circuit when that force has ceased to act; and, finally, that there is an elastic connection between the molecules, or that they are themselves elastic, in the same manner, as is the case with all solids subjected to bending, inasmuch as the molecules of such bodies are

then, within certain definite limits, stretched or pressed together on each side of the neutral axis, without this influencing either their connection or their relative positions. Now if, on the molecules being deflected by magnetic force, the relation between them is similar to that which exists when they are stretched or pressed together by mechanical force, there must be a certain reciprocal mobility between them, involving no change, however, in their *relative* positions, so long as it does not exceed a definite limit; and if, too, the molecules are conceived as exerting a resistance to such deflection, which increases with the magnitude of the latter, i. e. if, while the deflection goes on, there is a continuous increase of tension, such tension must sooner or later overcome the coercive power, after which the molecules are compelled to slide one upon the other, the tension henceforth continuing constant, or its increase at least being proportional to that of the coercive power. This power is, we know, but weak, so long as the molecular disturbance equals zero, increasing, however, with the angle of deflection.

Unless we exclude altogether the idea of elasticity existing in the molecules themselves or in their connection one with the other, — as well when angularly deflected as when stretched or pressed together, — then, in face of the indisputable fact, that a coercive power actually exists in hardened steel, the question is not *whether* a phenomenon such as that described *would* result, but to what extent it does exist; for, assuming one force to produce a deflection which is counteracted by another, and the molecules themselves or the connection between them to be elastic, they must obviously in a measure yield to the action of the opposing forces, thus giving rise to a stress which increases with the stretching or pressing together of the molecules and tends to force them back to their normal state. Now such a force counteracting the deflecting force is the coercive power, which, till the stress has reached a definite limit, does not permit of any change in the relative positions of the molecules, but which, on that limit being attained, yields to the superior force, allowing the molecules, to slide one

upon the other, until the latter force is connterbalanced by the former.

The resistance exerted by the coercive power is comparatively great when the degree of deflection is not very small, a fact which various phenomena tend to prove, whereas the tendency of the molecules to regain their original neutral state is in comparison but a trifling force in a closed circuit, in which the terminal surfaces of the magnet are connected by an armature or in some other manner, whereby the reciprocal saturation of all the layers is approximately effected. This tendency therefore cannot overcome the coercive power after the deflecting force has ceased to act, and consequently the molecules continue in the position given them by the deflection, only the stress spoken of above, forces their axes as far back as their elasticity has allowed of their being drawn out of their neutral position. Assuming the relation to be as here described, the phenomena observed during the experiments would obviously be a necessary consequence.

The following are the most important of these phenomena.

1. On the unmagnetized bar being placed in the electro-magnet, and the current for the first time closed, the deflection was, comparatively speaking, great, for there being as yet no molecular deflection the resistance arising from that cause was the least possible, and the instantaneous deflection not only comprised that which could exist within the limit of elasticity, but it passed far beyond it, and the molecules were therefore compelled to slide one upon the other, thereby suffering a permanent change in their relative axial position.

2. On the current being interrupted and the deflecting force ceasing to act, the molecules freed themselves from the state of tension in which, owing to their elasticity, they had been held by the deflecting force, at once springing back through an angle equal to that part of the previons deflection, which lay whithin the limit of elasticity.

3. Before the latter deflection took place, the galvanometer had been brought to rest, and the point of zero in the mirror had

settled on the vertical filament of the telescope. It is not impossible that during this interval the molecular deflection continued to increase to some extent with the continued action of the current, but at a rate so gradual as to have had no influence upon the galvanometer; if so the increase must of course have been in *permanent* deflection only. And it was therefore natural that when the current, after having been interrupted, was again closed, the instantaneous deflection should be confined within the limit of elasticity and be one in magnitude with the rebound. The deflection of the galvanometer proved such to be the case, for on closing the current again, the indication was equal in magnitude, though in a contrary direction, to that which took place on the current being broken.

4. On the magnitizing force being increased by connecting the electro-magnet with four cells, the first effect on the unmagnetized bar was far greater than in the first series of experiments with one cell only. But the effect consisted chiefly in an augmentation of the *permanent* deflection, for the deviation of the galvanometer, every time the current was subsequently interrupted and closed, indicated that the limit of elasticity had undergone little or no modification. In the third series, the magnetomotoric power of the electro-magnet had been further increased, and on the bar, — already highly magnetized, — being subjected to its action, there was again, the first time the current was closed, an increase in the permanent molecular deflection; but this having before been considerable, not only was the deflecting force (which is proportional to the magnetomotoric force multiplied by the *cosine* of angle of deflection) compared with the intensity of the current, less, — but the action of the molecular motion on the induction coil was also proportionately less so that the result, considering the intensity of the current, proved also less than before. The magnetizing force having been once more increased, this time by moving the bar lower down the electro-magnet, there was again a augmentation of the permanent deflection.

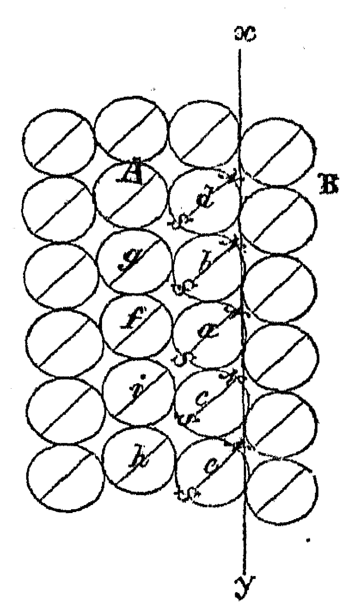
The fact of the deflection within the limit of elasticity, though increasing in the subsequent series of experiments as compared

with the previous ones, owing to the increase in the coercive power, yet, at the same time, causing smaller deviations of the galvanometer, is of course accounted for by the decreasing rate, at which the *sines* increase with the respective angles.

From what has been stated above, it would seem clear that, supposing the molecules to be themselves elastic, or an elastic connection to exist between them, phenomena such as those observed, would be a necessary result, and that we are justified in assuming the molecular positions in steel magnets to be subjected to changes of *two* essentially different kinds, one of which is *transient* and confined within certain definite limits of elasticity, the other *permanent*, taking place only when the elastic tension has increased so far as to overcome the coercive resistance, the molecules then sliding one against the other and assuming relatively new axial positions, in which they continue till the limit of the transient change is again passed in one direction or the other.

As the molecules after an *increased* deflection within the limit of elasticity seek to free themselves from their state of tension, by returning to the normal position for the permanent deflection, so likewise, after a *diminished* deflection will they strive to regain that position.

If in the figure below, *A* and *B* be conceived to represent a collection of molecules in a magnet, with a mean permanent deflection, represented by *NS*, in which they consequently saturate or counterbalance one another, and if the division *B*, to the right of the vertical line x y, be removed from *A*, the equilibrium of the molecules in both *A* and *B*, will no longer exist. In *A*, the north-pole magnetism in the terminal molecules, which before was saturated by the molecules in *B*, becomes free, and has now to seek saturation exclusively from the



molecules of A. If we consider more closely one of these terminal molecules a, with regard to the consequent mutual action on the other molecules, it will appear that its north pole is repelled with equal energy — at the same distance and the same angle with its axis — by the north poles of the molecules b and c, d and e &c., but in contrary direction, the result being therefore zero. But the south poles of these last mentioned molecules, on the other hand, do not counteract one another in their action on the north pole (N) of a, both on account of the difference in the distances and in the angles at which they act upon N; b's south pole is not only nearer to N than all the other poles, but its attracting influence is exerted at a much more favourable angle and in a direction tending to give NS a more vertical position. In the same manner, but with less effect, chiefly owing to the greater distance, d's south pole seeks to diminish the deflection of a; the south poles of c and e, on the other hand, deflect in a contrary direction, but with much less effect, owing both to the greater distance and to the unfavourable angle at which they act; their influence is consequently but trifling compared with that of the former and does not counterbalance it.

As the action of the south poles of these molecules on the north pole of a, chiefly consists in effecting a diminished deflection, their north poles exert an equally powerful influence in deflecting the south pole of a in the same direction.

With regard to f, g, h and i, in the next molecular layer, the difference between their attracting and repelling power is comparatively small, and their greater distance still further diminishes the small force with which N acts upon and is in turn acted upon by them. In this layer, f exerts the greatest influence and seeks to increase the deflection of a. The action of the remaining molecules need hardly be taken into account; for with some of them it is almost equal from both poles, with others the direction of the force is nearly parallel to the direction of the axis of a, and finally the deflecting power is counterbalanced in most, acting as they do in contrary directions.

Still less effect have the molecules in the third and following layers, on the deflection of a. This appears from an inspection of the figure.

It is evident, therefore, from what has been stated above, that, on the separation of B from A, the equilibrium existing between the molecules is at once put an end to. By the magnetism set free in the molecules of the terminal layers, these molecules act upon each other and upon those of the adjacent layers, in a manner, the result of which is a strong tendency to decrease the deflection of all molecules in the terminal layer. Irrespective of the direct action mentioned, the molecular disturbance in the terminal layer gives rise to a corresponding deflection in the adjacent layers. Thus, if a experiences a diminished axial deflection, its north pole approaches the north poles of f and g, and its south pole is drawn away from the south poles of those molecules; the repulsion of the north poles will consequently become greater than the corresponding repulsion of the south poles, and, therefore, seek to give f and g a deflection parallel to a. Now, the coercive power striving to counteract the sliding of the molecules one upon the other in their endeavours to take up new positions, the deflection taking place, must at first at least, depend on the elasticity, giving rise to a tension that will extend through the entire magnet to its opposite end.

Should the former conditions for saturation from the B division again arise, and no change meanwhile have taken place in the molecular grouping, this tension will force back the axes of the molecules into their former positions, provided no new circumstances arise to prevent it, and thus equilibrium will be restored.

The force with which the molecules strive to attain a less deflected position under the circumstances mentioned, is of course dependent on their angle of deflection, or on the magnetic intensity. Even when a body is but moderately magnetized, it will, to a greater or less extent, overcome the coercive resistance and permanently diminish the angle of deflection nearest the end where magnetism has become free.

If two homogeneous steel bars, of equal size and shape, be placed together, end to end, and magnetized as *one* bar, there will be no trace of free magnetism in the middle along the line of junction. On separating the bars, the molecular deflection experiences, as stated, a retrograde movement; the terminal surfaces before united, and the sides nearest them, where there was no free magnetism, now become magnetized. On again uniting the bars, the former conditions *may* possibly assert themselves, but a wholly different molecular arrangement may also be found to have been established. If the bars in contact were but slightly magnetized, and if consequently when the separation took place, the rebound of the molecules did not exceed the limit of elasticity, the original conditions are completely restored; but if the magnetic intensity — and a high degree thereof is not required — be sufficient to cause the limit of elasticity to be exceeded during the rebound of the molecules, a permanent diminution of the angle of deflection is the result; and if the bars are then again united, positive free magnetism will be observed to exist on the side of one, and negative on the side of the other, where the molecular sliding has taken place, — a wholly different state being thus established.

But this is not the only cause, — although the ordinary one, — which sometimes, and even generally, prevents the reestablishment of the original conditions after having once been disturbed; there is also another, namely, that dependent on the insufficiency of the force to restore the original neutral positions of the molecules. The molecules exert, by reason of their inertia, a resistance to the motion. It will afterwards be shown that this inertia, even in the softest iron, may be sufficient, when the energy of the magnetic forces in action is small, in a relatively considerable degree, to prevent the molecules from taking up the positions, which those forces seek to give them. On the application of external means, however, — slight shocks for instance — they are better enabled to yield to the force.

Hence there are two wholly different circumstances which can prevent the molecules — when again subjected to the action of

the same forces as before — from reaching the former angle of deflection, after that angle has been altered. But, when the molecules are uninfluenced by these two circumstances, there is every reason to believe that they always regain their original positions on being subjected to the action of the same forces; at least my numerous measurements of the relation of the moments in magnets have never led me to doubt this; and it will be shown in the sequel, by experiments specially conducted to elucidate this question, that the results fully justify such an assumption.

Now if a bar of homogeneous steel, formed into the shape of a ring, be acted upon, along its entire length, by some magnetic force, and the molecules be polarized in the direction of the length of the bar, the molecular magnets of each succeeding stratum will saturate each other, and no trace of free magnetism will be observed on the surface, whatever may be the extent of the polarization or molecular deflection. But if the ring be cut diametrically into two halves, the relation will be instantly changed on their being moved away from each other. The moments will be everywhere diminished, but chiefly at the ends, and their relative distribution throughout the length of either of the annular segments may be represented by a curve S C d d d D N, Fig. 1, Plate I.

So long as the ring remained closed and the molecules of every layer were acted upon equally from both sides, the deflection of the axes of the molecules was that determined by the magnetization, and it was the same in every layer. But, on the halves of the ring being separated and the terminal layers moved away from one another, one pole only of the molecular magnets in these layers could be saturated by the molecules behind, the other pole remaining to a greater or less extent unsaturated. While seeking saturation from the opposite poles of the adjacent molecules in the same stratum, the axes must be deflected backwards and take up a position forming a greater angle with the axis of the magnetic bar or annular segment than before. But, such a deflection of the molecular axes of the terminal layer cannot be effected without causing a corresponding deflection of the

axes in the layers behind; and this deflection will then be propagated in a rapidly decreasing proportion to the opposite end of the bar, where — if the original positions of the axes are conceived as being retained — the effect would be zero. Thus, let *S N*, Fig. 1, Plate 1 be the length of the half-ring, and *AS*, *ae*, *ae*, *ae*, *BN* equal to the moments for all the sections, previous to the division, then if, on the division taking place, the deflection or moment in *N* (equal to *NB*) be retained, while the molecular axes in all other parts are allowed to take up positions corresponding to the new conditions of equilibrium after the ring has been cut through, then the deflection will be every where diminished and to an extent equal to the ordinates *ab*, *ab*, *ab*, . . . *ab*, according to which the moments of the layers can be represented by the figure *SCBN*. But the molecules in the terminal layer at *N*, having in reality also been free, and having, as in *S*, sought to take up the new positions of equilibrium determined by their also being no longer acted upon in one direction by their former adjacent layers, a diminution of the deflection from *N* to *S* has also taken place equal to *cd*, *cd*, . . . *cd*, according to which the relative distribution of the moments along the length of the bar can be represented by the figure *S*, *C*, *d*, *d*, *d*, *D*, *N*, — a distribution common to all magnets with free ends.

This difference of moment in the succeeding layers gives rise to free magnetism on the side of the bar, save at the point *E*, where the difference is zero.

If the two halves of the ring are again united, the original distribution will be restored, provided no permanent change has, meanwhile, taken place in the relative positions of the molecules. The molecules in *S* and *N* will be saturated as before, and, together with the remaining molecules, become enabled to rebound and take up the same relative positions they had previous to the division of the ring.

A similar result will ensue if, instead of both the terminal surfaces of each half-ring, one only in each half be brought into contact with the other, or if the opposite poles of two

straight magnets of the same sectional area, length, and magnetic intensity be placed in contact with each other; in this case, however, the effect will only proceed from the two ends in contact, while scarcely a trace of it will appear at the free ends. If therefore the opposite poles of the two halves of the ring be placed in contact with each other in N, the backward deflection of the molecules will be greatest at that point, and the moment will be represented by the line NB. On the magnets subsequently being moved apart or brought together, the angle of the molecular axes will be changed accordingly and consequently the moment resulting therefrom will vibrate between the limits ND and NB. This cannot arise from the magnet having received and afterwards lost a corresponding degree of magnetism; for, as before stated, on the closed ring being magnetized, an equal axial deflection was conferred on the molecules in every layer, whereby each of these molecules could be saturated in equal degree from the two adjacent and equally deflected molecules on both sides, and they were thus enabled to retain the position that had once been given them. On one of these adjacent layers together with all the following layers being withdrawn, the former conditions of equilibrium ceased to exist, — one pole of the molecules in the terminal layer became free and unsaturated, and in striving to take up a new position of equilibrium, in which partial saturation of the poles could be effected, the deflections of the molecules, or their moments, were diminished, without in any way disturbing their relation one to another, provided the backward deflection did not exceed the limits of the transient variability. But the axial movement having taken place within the limits of molecular elasticity, a strain of the molecules has been the natural result, which strain may, therefore, be considered as a substitution for the diminution of the moment within the limits of elasticity. Therefore on the distribution of the forces again becoming restored, allowing saturation to take place in the original and, for the degree of magnetization, more natural position of the axes, the molecules, freeing themselves from

the strain, immediately spring back, thus causing the moment to be increased to its original magnitude.

It is just this which occurs, on the replacement of the two half-rings, magnetized to precisely the same degree. The axes then regain their original direction, provided of course no partial displacements have taken place in the positions of the molecules, which would prevent their doing so to the full extent.

But as the molecules in the closed ring, previous to its being cut through, cannot be conceived as magnetizing one another, as they are only by their reciprocal saturation enabled to retain the degree of magnetization by other means imparted to them, so neither can it be considered that the two equally magnetized halves of the ring, or their molecules, on again being brought into contact, after having been separated, magnetize each other. The mutual induction exerted, is simply a re-establishment of the original magnetic condition, which to some extent has resistingly been subjected to a transient change. In like manner the molecules can hardly be considered as suffering any loss in the degree of magnetization from their moments having been transiently diminished, in order to attain saturation, since for this diminution there has been substituted a force which repels them into their original position, as soon as saturation in it be possible.

I have dwelt at considerable length on the questions discussed above, that the reader may more clearly apprehend the sequel, from which it will appear that, assuming the relation to be as here set forth, a magnet, or magnet and keeper as a system, always represents a definite, unchangeable force, however different may be the distribution of that force in the several parts of the magnet and keeper under varying influences, and that the difference between the sum of the free magnetism in the several parts of an unarmed magnet, and the sum of the free magnetism as a whole in a magnet and keeper in contact, — a difference always constituting a comparatively minor matter, — is solely dependant on the *transient* deflection of the molecules and not on any change in the degree of magnetization or in their permanent deflection.

The restoration in the end-surfaces of the positions originally given by the deflecting force to the molecular axes, when equally powerful magnets are brought into contact with each other, can only occur in the event of the limit of elasticity, or the transient mobility, not having been exceeded during the retrogression at the time the terminal surfaces were first separated. This however presupposes the original magnetization of the bar to have been weak. If powerfully magnetized and if the angle formed by the molecular axes with the sectional plane of the magnet is large, the force by which the free ends of the molecules are deflected down towards the surface of the exposed end in order to attain partial saturation, and the angle they describe, are so great, that the limit of elasticity is always exceeded, and on the corresponding end of the equally powerful magnet being then replaced, the axes do not regain their former positions. If the magnets on being brought into contact imparted any additional magnetism to each other, it would be different; but this not being the case, the molecular axes only spring back till their elastic strain is spent, thus falling short of their original position by an angle, equal to that through which the molecules slided when the separation took place. As the moments consequently after such sliding are, on a portion of the magnet, decreasing towards the surface of contact and are likewise, on the other side of the point of indifference at the middle of the bar, decreasing towards the free end, both kinds of free magnetism will now be found diffused over the sides, whereas in the case first mentioned, — that in which the limit of elasticity was not exceeded, — there is, during the contact of the magnets, only north-magnetism on the side and free end of one, and south-magnetism on the side and free end of the other magnet, no free magnetism being present close to the surfaces in contact.

Measurements of the moments in a weakly magnetized bar will be found in Tabe 1 pag. 24, and are also represented in Pl. I, Fig. 1. In Fig. 2, Plate I, drawn on the same scale as Fig. 1, *S a b c N* shows the curve for a strongly magnetized bar with

the strain, immediately spring back, thus causing the moment to be increased to its original magnitude.

It is just this which occurs, on the replacement of the two half-rings, magnetized to precisely the same degree. The axes then regain their original direction, provided of course no partial displacements have taken place in the positions of the molecules, which would prevent their doing so to the full extent.

But as the molecules in the closed ring, previous to its being cut through, cannot be conceived as magnetizing one another, as they are only by their reciprocal saturation enabled to retain the degree of magnetization by other means imparted to them, so neither can it be considered that the two equally magnetized halves of the ring, or their molecules, on again being brought into contact, after having been separated, magnetize each other. The mutual induction exerted, is simply a re-establishment of the original magnetic condition, which to some extent has resistingly been subjected to a transient change. In like manner the molecules can hardly be considered as suffering any loss in the degree of magnetization from their moments having been transiently diminished, in order to attain saturation, since for this diminution there has been substituted a force which repels them into their original position, as soon as saturation in it be possible.

I have dwelt at considerable length on the questions discussed above, that the reader may more clearly apprehend the sequel, from which it will appear that, assuming the relation to be as here set forth, a magnet, or magnet and keeper as a system, always represents a definite, unchangeable force, however different may be the distribution of that force in the several parts of the magnet and keeper under varying influences, and that the difference between the sum of the free magnetism in the several parts of an unarmed magnet, and the sum of the free magnetism as a whole in a magnet and keeper in contact, — a difference always constituting a comparatively minor matter, — is solely dependant on the *transient* deflection of the molecules and not on any change in the degree of magnetization or in their permanent deflection.

on separating the two halves of the magnetic ring; the molecular poles of the free end of the keeper can no longer retain their former position, and in order to attain a greater or less degree of saturation, the axes recede, forming a smaller angle with the end-surface, and this backward deflection is propagated throughout the whole of the keeper and magnet to the farthest end of the latter. In such a keeper, with one end free and the other in contact with the magnet, two opposing forces are in action, viz. that proceeding from the magnet, which, in its attempt to attain a more complete saturation, strives to polarize the keeper, and that proceeding from the free end-surface of the latter, which seeks to counteract such polarization. By reason of the reciprocal counteraction of these forces, a tension arises in all parts of the keeper, which in the free end retains the molecules in a position, in which one of their poles is prevented from being wholly saturated by the adjacent molecules, which throughout the entire length of the keeper causes a difference in deflection, — hence a partially unsaturated state in all the molecular layers, — and which finally, in the terminal surface in contact with the magnet, opposes the complete saturation of the latter, thus likewise preventing the molecular axes there from attaining the degree of deflection corresponding to the magnetization, or that which they would be enabled to attain were they in contact with another magnet of equal power.

This diminution of the deflection of the molecular axes nearest the keeper, or diminution of the magnetic moment there, entails a corresponding diminution throughout the entire length of the magnet, nearly proportional to that which results on the keeper being removed.

Now it is evident that, if there were no resistance to the propagation of the magnetizing force, the stress would be nearly in inverse proportion to the length of the keeper, since the number of the imaginary transverse strata increases with the length, and, for a certain degree of general deflection, the magnitude of its difference in the several strata — on which the stress depends — is of course inversely proportional to the number of these strata

free ends, of the same dimensions as that represented in Fig. 1. S, a, n, n, i N represents the moments when another magnet of the same power and dimensions is brought into contact with the N-end. S, a, k, m, m, l, N would have represented these moments, had not the limit of elasticity been exceeded in the movements of the axes throughout that part of the bar where the backward deflection was greatest when the ends became free, and carried the molecules along with them in the direction indicated by the ordinates mn, mn, li. Measurements of the moments as represented in the figure will be found in Table 3, IV pag. 25.

If such a magnet be suffered to remain for any length of time with exposed ends, the stress in the successive transverse layers resulting from their unequal deflection and consequent incapacity to saturate one another, causes a general retrogression of the moments, chiefly where the difference of deflection and the stress resulting from it is greatest, whereby that difference is gradually diminished, and the relation between the moments at the several parts of the bar becomes what it would have been at first, had not the limit of elasticity been exceeded. Hence, if two magnets thus weakened and of equal power be brought into contact, the magnitude of their moments will continue to increase up to the ends in contact, and consequently there will be but one kind of free magnetism diffused over the surfaces of each of them. With regard to the distribution of the moments in a magnet thus weakened, with free ends as well as in contact with a magnet of equal power, and with a keeper under different circumstances, *vide* tables 17 and 18, pag. 31.

If a bar of soft iron, not too long, be inserted between two magnets, the result, so far as the magnets are concerned, will be much the same as that which ensued on their being placed in direct contact, for all the terminal layers in contact as well as all the intermediate strata in the iron bar or keeper will then very nearly saturate one another, so that but little free magnetism will be traced on the side of the keeper. But if one of the magnets be moved away from the keeper, the result will be somewhat similar to that which ensued

on separating the two halves of the magnetic ring; the molecular poles of the free end of the keeper can no longer retain their former position, and in order to attain a greater or less degree of saturation, the axes recede, forming a smaller angle with the end-surface, and this backward deflection is propagated throughout the whole of the keeper and magnet to the farthest end of the latter. In such a keeper, with one end free and the other in contact with the magnet, two opposing forces are in action, viz. that proceeding from the magnet, which, in its attempt to attain a more complete saturation, strives to polarize the keeper, and that proceeding from the free end-surface of the latter, which seeks to counteract such polarization. By reason of the reciprocal counteraction of these forces, a tension arises in all parts of the keeper, which in the free end retains the molecules in a position, in which one of their poles is prevented from being wholly saturated by the adjacent molecules, which throughout the entire length of the keeper causes a difference in deflection, — hence a partially unsaturated state in all the molecular layers, — and which finally, in the terminal surface in contact with the magnet, opposes the complete saturation of the latter, thus likewise preventing the molecular axes there from attaining the degree of deflection corresponding to the magnetization, or that which they would be enabled to attain were they in contact with another magnet of equal power.

This diminution of the deflection of the molecular axes nearest the keeper, or diminution of the magnetic moment there, entails a corresponding diminution throughout the entire length of the magnet, nearly proportional to that which results on the keeper being removed.

Now it is evident that, if there were no resistance to the propagation of the magnetizing force, the stress would be nearly in inverse proportion to the length of the keeper, since the number of the imaginary transverse strata increases with the length, and, for a certain degree of general deflection, the magnitude of its difference in the several strata — on which the stress depends — is of course inversely proportional to the number of these strata

over which it is distributed. And as it is this tension which causes even the extreme free end to become to a certain extent polarized, the longer the keeper, the *less* will be the moment of the free end-surface, and the less the free magnetism on that surface.

As, on the other hand, this stress also tends to counteract the deflection in the terminal layer in contact with the magnet and therefore its capacity to saturate the latter, the longer the keeper, the *more* deflected will be the molecules on the surface of contact, and the more fully will it saturate the magnet, i. e. the greater will be the moment both in the magnet and keeper in the surface of contact. This stress in the keeper, however, is dependent not only on the length of the keeper, but also on its hardness. In the softest iron, even, there is a certain resistance to the magnetic induction, and the harder it is, the greater is the resistance which prevents the propagation of the magnetizing force. In keepers of equal dimensions, therefore, the action of the magnet decreases more rapidly when they are of hard than when they are of soft iron; and the more distant parts of hard keepers, being thus hindered from participating to the same extent as those of the soft ones in the deflection to which the contact-surface is subjected, the greatest deflection and consequently the capacity of the keeper to saturate the magnet, cannot be so great in hard as in soft keepers.

What is stated here is evident from the accompanying series of observations of the distribution of the moments in keepers of different lengths and degrees of hardness, and from the graphic representation thereof in Plate 1 Fig. 2, in which *SN* represents the length of the magnet A, which shortly before the measurements were undertaken, had been strongly magnetized. *S a b c N* is, as before stated, the magnetic moments, when the magnet is free and unarmed (*vide* Tab. 3 I. pag. 25.) *S a d c N* represents the moments, when the north end of the magnet is in contact with one end of the keeper **a**, of very soft iron, and of the same dimensions as the magnet (length 146^{mm}, diam. 10.5^{mm}), the other end being free

(Tab. 3, II). $N\ e\ \alpha\ E$ is then the moment of the keeper (Tab. 4 *a* pag. 26).

$S\ a\ f\ g\ N$ represents the moments of the magnet, when the south end of the magnet B, of the same strength and dimensions as A, is placed in contact with the far end of **a** (Tab. 3, III); $N\ g\ \beta\ E$ is then the moments of the keeper (Tab. 9). $S\ a\ n\ n\ i\ N$ gives the moments of A when the south end of B is in contact with its north end (Tab. 3 IV).

$N\ \gamma\ \delta\ D$ represents the moments of the keeper **d**, length 45^{mm}, diam. 10.5^{mm} (Tab. 10), and $N\ \varepsilon\ \zeta\ F$ the moments of the keeper **e**, length 292^{mm}, diam. 10.5^{mm} (Tab. 11) when these two keepers are inserted between the north end of A and south end of B.

$N\ \eta\ \theta\ E$ gives the moments of the keeper **b**, of the same dimensions as **a**, but harder (Tab. 4 *b*); $N\ \iota\ \varkappa$ A the moments of the keeper **c**, of the same dimensions but harder still (Tab. 4 *c*); $N\ \lambda\ \mu\ E$ the moments of a soft steel-bar, also of the same dimensions as **a**, **b** and **c** (Tab. 4 *g*). $N\ \nu\ \xi\ \varphi$ gives the moments of the keeper **f**, length 73^{mm} and of the same diameter as the rest (Tab. 6); and $N\ o\ \pi\ D$ the moments of the keeper **d**, length 45^{mm} (Tab. 5) — all these keepers with one of their ends in contact with A and with the other free. Finally, $N\ \rho\ \sigma\ F$ gives the moments of the keeper **e**, length 292^{mm} (Tab. 7), and $N\ \rho\ \tau\ G$ that of **e** with which **b** is in contact, so that together they form a keeper 438^{mm} long with a diameter of 10.5^{mm} (Tab. 8), when both these keepers (**e** and **e** + **b**), as the foregoing, have one of their ends in contact with A, and the other free.

The keepers **b**, **d**, **e**, and **f** are of the same degree of hardness, and lie, in that respect, between **a** and **c**.

It will be seen from the curves for the keepers **b**, **d**, **e** and **f**, that the shorter the keeper, the more rapidly do its moments decrease from the magnet towards the free end, and further, the greater is the moment in that free end, and the less in the opposite end. Hence the shorter the keeper, the less will the successive strata saturate each other; the more intense will be the free

magnetism which is distributed over the side and on the terminal surface furthest from the magnet; the less able will be the keeper to saturate the magnet, and in consequence the greater will be the tension between the molecules. The cause of this is evident from what has been already stated, and need not therefore be pointed out.

With regard to the free magnetism, it is, in the different parts along the side, proportional to the dip of the tangents to the corresponding points of the curve of moments, and with regard to the magnetism collected on the terminal surfaces, it is as will afterwards be shown, proportional to the terminal ordinates of the curves; hence the free unsaturated magnetism on the side and on the terminal surface furthest from the magnet is more intense, though less in quantity, in short than in long keepers, and the magnetism on the surface of contact saturated by the magnet, is greater in long than in short keepers.

The figures further show that the harder the keeper, the greater is the resistance offered to the propagation of the magnetizing force; the more rapidly do the moments nearest the magnet decrease, and the less able is the keeper to saturate the latter. The keepers a b c and g are all of the same dimensions, but the three first are of iron of different degrees of hardness, and the last is of soft steel. The resistance offered to the magnetic induction thus increasing with the hardness, — the harder the keeper, the less is the stress, and the deflection depending on it, in the parts remote from the magnet, and hence the less also the magnetism of the free end-surface and the side near it. All this is clearly represented by the curves, Fig. 2.

It has been stated already that the ability of saturating the magnet increases with the length of the keeper; but, as will be seen from the figure, this property is easily counterbalanced by the diminution in the saturating power produced by the greater resistance to the magnetic propagation in virtue of the hardness of the iron. The keepers **e** and (**e** + **b**), the

former 292^{mm} and the latter 438^{mm} long, are, by reason of their greater length, better able to saturate the magnet than all the other keepers of the same degree of hardness; but the keeper a, only 146^{mm} long, being softer, nevertheless saturates the magnet more than either of the former, one of which is twice and the other three times as long.

When a keeper is placed between the like poles of two magnets, it is polarized in opposite directions towards both magnets. If these magnets are equally powerful, the moment in the middle of the keeper is zero, the two equal opposing forces counterbalancing each other there. A keeper, if not very long and acted upon in this manner, will have a considerable quantity of free magnetism of one kind almost equally distributed over the whole of its side. If the two magnets are of unequal power, the point where the moments equal zero is nearer to the weaker than to the stronger magnet, and the free magnetism on the surface will be of different intensity on both sides of that point. *Vide* Tables 12 and 13, giving the moments, and Plate 1, Fig. 2, $N\ \upsilon\ \varphi\ \chi\ E$ and $N\ \psi\ \varphi'\ \omega\ E$.

It is evident from what is now stated, that as a general rule, when a pole-end of a magnet is brought into contact with soft iron, it induces in the latter, on the surface of contact, a quantity of magnetism of the opposite kind to that of its own, which, in saturating the free magnetism of the pole, is itself in return saturated and ceases therefore to be free, — while the corresponding quantity of the opposite magnetism (the same as that saturated in the magnet) is distributed over the surface of the iron. Thus if the pole of the magnet be placed against the *side* of the keeper, free magnetism of the same kind as that of the magnetic pole-end will, except at the point of contact, be distributed all over the side as well as over the terminal surfaces of the keeper. Table 14 gives the moments in such a keeper, when the point of contact is in the middle, and Table 15 and 16, when both arms of the keeper are to each other respectively as 1 : 2

and as 1 : 4. A graphic representation of this relation will be found in Fig.s 3, 4 and 5.

Observations with a low degree of magnetism.

1.

Deviations of Galvanometer for the moments in a magnet A, diam. 10.5^{mm}, length 146^{mm}.

I, when both ends of the magnet are free; II, when the north polar-end is in contact with an armature of the same dimensions, the further end of which is free; III, when the north polar-end is in contact with an armature, the further end of which is in contact with the south polar-end of magnet B of the same dimensions and magnetic intensity as A; IV, when the north polar-end is in contact with the south polar-end of B.

Distance in millimeters	Deviations			
	I	II	III	IV
5.5 fr. *N*-end	2.40	6.20	7.35	8.08
15	3.63	6.60	7.57	8.05
25	4.58	6.93	7.62	8.01
35	5.35	7.15	7.68	7.95
50	5.98	7.23	7.63	7.82
65	6.20	7.18	7.47	7.62
65 fr. *S*-end	6.18	6.84	7.06	7.17
50	5.75	6.22	6.39	6.48
35	4.93	5.32	5.46	5.53
25	4.10	4.38	4.47	4.52
15	3.26	3.35	3.38	3.40
5.5	2.02	2.10	2.12	2.12

2.

Deviations for moments in the armature in contact with magnet. I, when the far end is free; II, when it is in contact with the south end of B.

Distance from end of oppos. magnet	Deviations	
	I	II
5.5mm	5.40	7.02
20	4.55	6.73
35	3.86	6.49
50	3.22	6.33
65	2.63	6.21
80	2.18	6.20
95	1.70	6.25
110	1.28	6.42
125	0.80	6.77
140.5	0.43	7.07

Observations with a high degree of magnetism.

3.

Deviations for moments in magnet A, diameter 10.5mm, length146mm.

I, when both ends of the magnet are free; II, when the north polar-end is in contact with the armature a, (*vide* below) the far end of which is free; III, when the north polar-end is in contact with a, the far end of which is in contact with the south polar-end of B, of the same dimensions and magnetic intensity as A; IV, when the north polar-end is in contact with the south polar-end of B.

Distance in millim.	Deviations			
	I	II	III	IV
5.5 fr. *N*-end	8.80	22.68	25.95	26.41
15	14.70	24.62	27.08	27.42
25	18.75	26.08	27.98	28.23
35	21.58	27.05	28.58	28.78
50	24.48	27.82	28.80	28.96
65	25.18	26.97	27.96	28.08
65 fr. *S*-end	24.25	25.22	25.98	26.08
50	22.27	22.96	23.51	23.57
35	19.32	19.82	20.25	20.29
25	16.72	17.20	17.37	17.40
15	13.12	13.44	13.55	13.56
5.5	8.00	8.19	8.25	8.25

4.

Deviations for moments in armatures of 10.5^{mm} diam. and 146^{mm} length, in contact with the magnet. **a** armature of very soft iron; **b** of harder do.; **c** of still harder do.; **g** of soft steel.

Dist. from end of opposite magnet	Deviations			
	a	b	c	g
5.5	20.43	18.40	17.85	11.38
20	17.86	16.00	15.46	8.84
35	15.40	13.78	13.19	6.86
50	13.12	11.72	11.11	5.12
65	11.07	9.77	9.27	4.00
80	9.05	8.00	7.56	3.03
95	7.07	6.25	5.88	2.23
110	5.22	4.60	4.28	1.53
125	3.58	3.03	2.78	1.01
140.5	1.80	1.52	1.42	0.57

5.

Deviations for moments in armature **d**, of same degree of hardness as **b**, diam. 10.5^{mm}, length 45^{mm}, in contact with magnet.

Distance	Deviations
5.5^{mm}	13.25
20.0	9.38
39.5	3.90

6.

Deviations for moments in armature **f**, of same degree of hardness as **b**, diam. 10.5^{mm}, length 73^{mm}, in contact with magnet.

Distance	Deviations
5.5^{mm}	15.25
20	12.07
35	9.05
50	6.09
67.5	2.95

7.

Deviations for moments in armature **e**, of same degree of hardness as **b**, diam. 10.5mm, length 292mm, in contact with magnet.

Distance	Deviation	Distance	Deviation
5.5mm	20.45	155mm	5.80
20	18.30	170	5.00
35	16.32	185	4.27
50	14.40	200	3.58
65	12.77	215	2.98
80	11.26	230	2.42
95	9.94	245	1.92
110	8.74	260	1.46
125	7.65	275	1.00
140	6.61	286.5	0.58

8.

Deviations for moments in armature (**e** + **b**), diam. 10.5mm, length 438mm, in contact with magnet.

Distance	Deviation	Distance	Deviation
5.5mm	20.50	200mm	3.90
20	18.36	215	3.33
35	16.39	230	2.80
50	14.48	245	2.34
65	12.86	260	1.91
80	11.37	275	1.50
95	10.07	290	1.13
110	8.89	320	9.70
125	7.82	350	0.43
140	6.81	380	0.26
255	6.03	410	0.15
170	5.26	432.5	0.10
185	4.56		

9.

Deviations for moments in armature a, in contact with the north polar-end of A and the south polar-end of B.

Distance	Deviation	Distance	Deviation
5.5^{mm}	25.18	80^{mm}	24.01
20	24.71	95	24.18
35	24.37	110	24.37
50	24.18	125	24.71
65	24.01	140.5	25.18

10.

Deviations for moments in armature d, in contact with the north polar-end of A and with the south polar-end of B.

Distance	Deviation
5.5^{mm}	24.38
22.5	24 28
39.5	24.38

11.

Deviations for moments in armature e, in contact with the north polar-end of A, and with the south polar-end of B.

Distance	Deviation	Distance	Deviation
5.5^{mm}	22.84	155^{mm}	15.55
20	21.17	170	15.66
35	19.78	185	15.96
50	18.64	200	16.38
65	17.70	215	16.96
80	16.96	230	17.70
95	16.38	245	18.64
110	15.96	260	19.78
125	15.66	275	21.17
140	15.55	286.5	22.84

12.

Deviations of the moments in armature a, in contact with the north polar-ends of A and B.

Distance	Deviation	Distance	Deviation
5.5^{mm}	16.31	80^{mm}	1.61
20	12.40	95	4.90
35	8.43	110	8.43
50	4.90	125	12.40
65	1.61	140.5	16.31

13.

Deviations for moment in armature a, in contact with north polar-ends of A and of the weaker magnet C.

Distance	Deviations	Distance	Deviations
5.5^{mm}	17.83	80^{mm}	4.54
20	14.80	95	2.30
35	12.00	110	0.07
50	9.35	125	3.28
65	6.86	140.5	6.90

14.

Deviations for moments in armature e, the middle of the *side* of which, is in contact with A.

Dist. from end of each arm.	Deviation	Dist. from end of each arm.	Deviation
5.5^{mm}	0.68	80^{mm}	5.70
20	1.60	95	6.75
35	2.57	110	7.65
50	3.60	125	8.50
65	4.67		

15.

Deviations for moments in armature **e**, the *side* of which, at $^1/_3$ of its length from one of the ends, is in contact with the magnet.

Dist. from end of longer arm	Deviation	Dist. from end of shorter arm	Deviation
5.5mm	0.54	5.5mm	1.35
20	1.20	20	2.98
35	1.95	35	4.22
50	2.81	50	5.75
65	3.65	65	6.62
80	4.62	80	7.11
95	5.58		
110	6.60		
125	7.68		
140	8.95		
155	10.03		
170	11.00		

16.

Deviations for moments in armature **e**, the *side* of which, at $^1/_5$ of its length from one of the ends, is in contact with the magnet.

Dist. from end of longer arm	Deviation	Dist. from end of shorter arm	Deviation
5.5mm	0.50	5.5mm	1.60
20	1.06	20	2.98
35	1.63	35	4.72
50	2.35		
65	3.00		
80	3.75		
95	4.58		
110	5.42		
125	6.46		
140	7.56		
155	8.82		
170	10.23		
185	11.50		
200	12.72		
215	13.77		

Observations with reduced magnetic intensity.

17.

Deviations for moments in magnet A.

I, when both ends of the magnet are free; II, when the north polar-end is in contact with the armature **a**, the far end of which is free; III, when the north polar-end is in contact with **a**, the far end of which is in contact with the south polar-end of the magnet B, of the same dimensions and magnetic intensity as A; IV, when the north polar-end is in contact with the south polar-end of B.

Distance in Millim.	Deviation			
	I	II	III	IV
5.5 fr. *N*-end	4.27	9.60	11.38	12.15
15	6.03	10.08	11.40	12.13
25	7.34	10.43	11.40	12.00
35	8.38	10.62	11.36	11.82
50	9.39	10.75	11.24	11.53
65	9.78	10.55	10.81	10.97
65 fr. *S*-end	9.55	10.00	10.17	10.26
50	8.77	9.07	9.16	9.22
35	7.52	7.60	7.66	7.69
25	6.60	6.75	6.78	6.80
15	5.52	5.68	5.70	5.70
5.5	4.28	4.36	4 37	4.37

18.

Deviations for moments in a keeper in contact with magnet.

I, when the far end is free; II, when it is in contact with the south polar-end of B.

Dist. from end of opposite magnet	Deviation	
	I	II
5.5^{mm}	8.64	11.20
20	7.60	10.86
35	6.52	10.63
50	5.53	10.48
65	4 58	10.40
80	3.70	10.40
95	2.87	10.48
110	2.10	10 63
125	1.36	10.86
140.5	0.65	11.20

II.

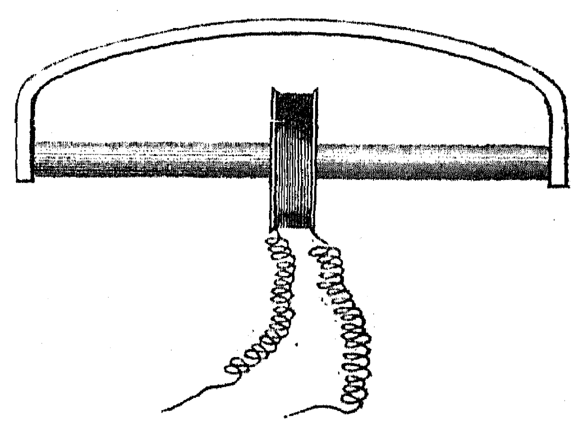
After the preceding general remarks on the magnetic distribution, when a magnet is in contact with an armature, or with another magnet of about the same intensity as its own, we are prepared for the examination of the main subject of enquiry, namely, of the conditions, which determine the relation existing between the magnetic attraction and the distance between the attracting bodies; — or, for the examination of the variations in the molecular axial positions that take place, whenever the distance separating magnetic bodies under the influence of each other undergoes any change.

It may, however, not be without interest, before proceeding to this part of our subject, to make a few observations on the state of the molecular moments in a magnet, during its magnetizing process, consequent on being brought under the influence of a magnet of preponderating intensity, as compared with its own, — although what may be here communicated on this subject will not be necessary for the comprehension of what follows, neither will it form anything like an exhaustive account of the molecular state of the magnet, during its various stages of magnetization, it will consist merely of isolated observations, made partly during the experiments undertaken with a view of ascertaining the nature of molecular motion, — as referred to before, — and partly during the experiments performed for the purpose of ascertaining the condition of the moments in the various parts of

the magnet, before the sudden diminution takes place in the molecular torsion, which occurs the instant that the end-surfaces become free.

During the progress of these experiments several phenomena of interest were observed which invited closer investigation; but this could not be carried out at the time, without preparations and without giving up the main object in view, — and since then I have not had leisure to recommence the enquiry.

In order to examine the relation between the molecular moments along the entire length of a magnet immediately after magnetization, and before the retrogression of the molecules had taken place, which arises in the terminal surfaces as soon as these become free, — the two magnets before named, A and B, were — after remaining with the induction-coil on for half an hour in the great electro-magnet of the University, under the influence of four large Bunsen-cells, — slid out from the electro-magnet direct into an iron-bow (see annexed figure), by means of which the end-surfaces were kept connected, as if by the application of an armature. By quickly moving the coil, which was connected with the galvanometer, between certain measured intervals of the magnets, from one end to the other, the difference between the magnitudes of the moments



was determined along the whole length of the magnets. As a graphic representation of the magnetic distribution will render this more clear, such will be found in Pl. II, Fig. 1; the numerical results of the observations being altogether left out.

The curves obtained for each of the magnets by the measurements are indicated by $a\ a'\ a$. The induction-coil was placed on the centre of the magnet, and the bow torn away; the galvanometer then gave deviations represented by the distances $a'\ c'$, after which the bow in each case was immediately replaced. The distribution of the moments was again measured in the same way as before, and the relation between them was now found to be as indicated

by the curves b b' b. After the coil had again been placed on the centre of the magnets, the bow was torn away a second time and the galvanometer gave deviations represented by the distances b' c'. As soon as the mirror had come to rest, the coil was quickly slid off the magnets, by which deviations equal to c' d' were obtained. The moments along the whole length of the magnets were then found to be as represented by the curves c c' c. If, after the bow had been torn away the *first* time, the coil had been slid off the bar, and had then caused the same amount of deviation, c' d', as it did on being slid off, after the bow was torn away the *second* time — then of course the moments of the bar would, immediately after the magnetization, have been *ad a'd' ad.* It is, however, probable that the deviation would have been somewhat larger than $c'd'$, and the moments of the bar consequently somewhat larger than *ad a'd' ad.* From the operations performed it follows directly, that the moments were equal to *bd 'b d' bd* immediately before the bow was torn away the second time.

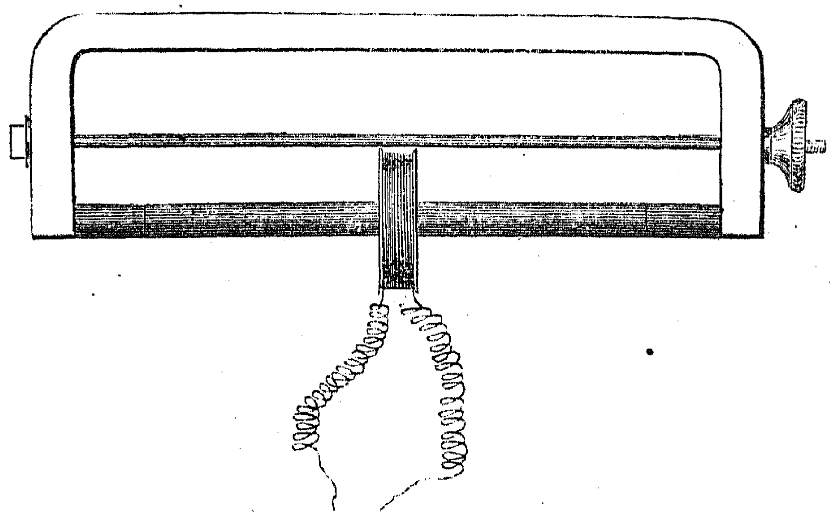
It may be mentioned that both the magnets used were made by Mr. Apps, of London, and that they eminently possess the quality belonging to good magnets, namely, that of retaining for a long time a high degree of magnetism, even when used in a manner which generally has a weakening effect.

It had previously been observed that B was always somewhat weaker than A, and that its point of indifference was situated at some distance from the centre. From the figure it will be plainly seen, that the cause of both these properties is the same; but the cause of the peculiar form of the curve a a' a, in this magnet, which reappears in a greater or less degree, or may be traced in the curves *bb'b* and *cc'c* last obtained, was not clear to me; it seemed likely that to a great extent at least, it depended upon unequal hardening, — such a condition imparting to steel an unequal resistance to the molecular torsion; but the deep depression in a a' a opposite the greatest convexity of c c' c, seemed also to indicate, that magnetization in a reverse direction had been previously performed, the effect of which had not been altogether

effaced by the latter magnetization. I therefore magnetized a bar of exactly the same dimensions, first in one direction and then in the other; and on ascertaining the differences between the moments while the bar was in the bow, and before the surfaces of the ends had been exposed, similar undulation was found in the curve connecting the ordinates representing the moments, and here, as in the former case, giving rise to the free magnetism along the side changing at several places from north to south. The power, with which the magnetization was effected was, however, not great in this case, being only that from three Meidinger-cells, applied to the great electro-magnet of the University. Before applying sufficient power for reversing the magnetism in the bar, it was demagnetized as far as possible, the bar, however, still retaining a slight degree of magnetism, of which the South was situated at *both* ends, while the greatest North polarity was a little more than one third the length from one of the ends.

I had been under the impression that the moments along the whole length of the magnet, immediately after magnetization and before the ends were at all free, would have differed as to their magnitudes in a far less degree than they did in A and B. In the results obtained — even for A, in which scarcely any irregularity seemed to exist in the nature of the steel, or at least no important want of symmetry with regard to the magnitude of the moments on both sides of the middle — these moments were nearly $^1/_3$ smaller at the ends, than at the middle. The reason might possibly be traced to the bows, the transverse areas of which were only about $^1/_3$ that of the magnet, and which therefore might exert a great resistance in the transfer of the magnetism from one end of the magnet to the other; and again the bows were far from fitting close to the end-surfaces, a circumstance which might considerably diminish the facility of saturating the magnetism of the ends. Finally, it did not seem improbable that the difference in the moments which existed since the previous magnetization, might, at least to some extent, have caused the unequal distribution, which

appeared now. A new steel-bar C was therefore made of the same dimensions as A and B, together with a bow which it fitted exactly and which formed a continuation of the bar for a distance of two centimeters at each end, and had a transverse area of about $1\frac{1}{2}$



times that of the steel-bar. With this the following series of experiments were made, in order to throw light on the fact mentioned, and at the same time to investigate what has previously been considered, concerning the mobility of the molecules.

The large electro-magnet of the University was, as before, made use of for magnetization, to which at first only one Bunsen-cell was applied. A preliminary examination was made of the effect of the electro-magnet on the galvanometer at about 8 meters distance from it, by opening and breaking the current; the effect proved not to be of sufficient importance to be taken into consideration. Next, the effect of the electro-magnet on the induction-coil alone was determined. For this purpose the coil was placed between two moveable iron-blocks belonging to the electro-magnet, and upon a graduated rule of wood, of the same dimensions as the likewise graduated steel-bar to be magnetized. While sliding the coil along the graduated rule, interval by interval, and opening and breaking the current, the effect on the galvanometer was observed; and by subtracting the deviations thus obtained, from those appearing when the same operations were performed with the coil placed on the steel-bar. the differences express the deviations corresponding to the effect on the steel-bar *alone*.

For such of the operations which produced strong inductive currents, an auxiliary wire-conductor, or shunt, was inserted, which led off the greater part of the current, and, as mentioned previously, only allowed $^1/_{13.9}$ of the current obtained, to pass through the galvanometer. In the following, all the noted deviations are reduced to correspond to the whole current-force, and graphically represented on Pl. II, Fig. 2, 3 and 4.

After the coil, fixed to the middle of the wooden rule, had been placed between the iron-blocks or armatures of the electro-magnet, the current from the electric element was made and broken several times, by which a deviation, averaging 6.53, was obtained. The moving of the coil step by step along the rule, gave deviations according to which the curve *a a a*, Pl. II, Fig. 2, is drawn; and the effect of the opening and breaking of the current upon the coil, when at the different distances from the poles of the electro-magnet, is therefore represented by the ordinates *ab*, *ab*, *ab*.

The steel-bar, with the coil on the middle, was now placed in the electro-magnet, and by making a current from the cell, the galvanometer gave a deviation equal to 112.9; but as 6.53 of this represents the effect on the coil itself, 106.4 corresponds to the effect on the steel-bar alone. The current was again interrupted, as soon as the mirror had come to rest, and gave then, as well as by a number of successive openings and interruptions of the current, deviations, which within slight and mere accidental variations, were always 42.46. When from this is deducted 6.53, as belonging to the effect on the coil, 35.93 remains, which represents the molecular oscillations, produced in the bar by the openings and interruptions of the galvanic current, — as mentioned p. 3.[1])

In order to judge of the magnitude of the deviations here

[1]) The deviations specified Pag. 3 are not reduced to the true current-force, they are merely those caused by that fraction of the current which passed through the galvanometer, and must consequently be multiplied by 13.9 to correspond to the total inductive current.

specified, it may be mentioned, that the greatest moment which can be retained at the middle of the free magnet-bar (without the bow) after being magnetized, probably by any power, is about 24.3, which is less than $^1/_4$ of the moment given to the molecules by the first shock of the magnetizing power from only one cell.

The differences between the moments along the bar, were now examined, while the current was in action, by moving the coil step by step along the whole length, while the deviations of the galvanometer were read off at each move. According to these deviations the curve *c c c* is drawn; but as *ab*, *ab*, *ab* have to be deducted from the moments belonging to this curve, being the effect on the induction-coil, the resultant curve *d d d* belongs to the magnet alone. The true lengths of the ordinates of this curve are uncertain, as it could not on this occasion be ascertained whether or to what extent, they may have been increased after the first electromotive shock had taken place, while the bar remained in the electromagnet. At all events they do not vary *more* than between the limits 109.2 and 105.9.

After the current was interrupted, the bar was slid from the electro-magnet direct into the bow, so that the end-surfaces were not exposed for one instant. It was intended during this operation to observe the deviation, which, it was presumed, would take place; but it appeared that the mere moving of the bar from one part to another within the electro-magnet, caused violent oscillations of the galvanometer, showing that the moments underwent material changes, even before the bar left the electro-magnet, — and no conclusion could therefore be drawn with regard to the decrease taking place in the moment. When the bar in the bow was removed from the electro-magnet, — an operation requiring no small force, for the electro-magnet held the bow tight, although the current was interrupted, — the magnetic moment naturally again became considerably diminished.

The differences between the moments along the bar, after it had been transferred to the bow, are represented by the curve *e e e e.* From its form it will be seen, that the difference between

the largest and smallest moments, is not great, as in the case represented in Fig. 1. (The position of this curve with regard to the scale on the drawing, has no reference to the real magnitude of the moments.)

By an accident the magnet escaped from the bow, so that the ends became free, and fearing that the consequent retrogression of the molecules at and towards the ends, might show its effect after the increased magnetization that the bar was afterwards to undergo, it was made red-hot and again hardened, before the following set of experiments was performed.

Four cells were connected with the electro-magnet, whose magnetomotive effect was about twice[1]) as great as that of one cell. The effect on the coil alone, placed between the poles, was equal to 11.26. As the steel-bar, with the coil fixed to the middle, was placed in the electro-magnet and the current passed, the galvanometer made a deviation equal to 116.8, of which — deducting the above 11.26 — 105.5 corresponds to the effect of the steel-bar alone. This is almost the same deviation as when only one cell was used. A slight motion of the bar was, however, observed the instant the current was opened, which proved to be caused by its not being at right angles to the side of one of the moveable iron blocks of the electro-magnet, between which it was placed, — the plane of one of the ends of the bar in consequence not fitting closely to the block. The position of the latter was therefore slightly altered after the current had been interrupted, and the bar was also lowered slightly; and on the current being again opened, the galvanometer

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[1]) The resistance in the coil of the electro-magnet is 6.1 that of a given length of wire, and the resistance in one Bunsen-cell = 3.3. The magnetomotive effect of n cells is therefore $i_n = \frac{n\,E}{3.3\,n + 6.1} = \frac{E}{3.3 + \frac{6.1}{n}} = i_1 \cdot \frac{2.85}{1 + \frac{1.85}{n}}$, E being the electromotive, and $i_1 \left(= \frac{E}{9.4}\right)$ the magnetomotive power of one cell. Thus the effect of 4 cells is = 1.95 times that of one cell.

gave a deviation of 207.1, of which 195.8 appertained to the effect on the bar. The deviation at the interruption of the current before the alteration in the position of the bar, was not observed.

On subsequently making and breaking the current several times, while the bar remained in the electro-magnet, the deviations were, within slight accidental variations, equal to 42.71, of which, after deducting 11.26 (the effect on the coil), 31.45 corresponded to the oscillations of the molecules.

An examination was now made, in the previous manner, of the differences between the moments along the bar, while still in the electro-magnet from which the current had been interrupted; they are represented by the curve *a a a* Fig. 3, Pl. II; likewise the differences between the moments, after the bar had been slid into the bow, by the curve *b b b*. The position of none of these curves, with regard to the scale opposite in the figure, has any reference to the real moments.

The number of cells was then increased to sixteen, united in series, zinc to carbon, as in the former case, — taking for granted, as I did without any examination of the resistances, that a vast increase of magnetic power would be obtained. On a subsequent examination being made, of the resistances, however, it appeared that the above combination was far from advantageous, — the magnetomotive effect obtained, being but 2.58 times that from one cell.

The influence of the current on the coil alone, could not be ascertained, as the latter was placed on the bar, and this fixed in the bow; but it may no doubt be considered, that, now the magnetomotive effect was one fourth greater than when four cells were used, it was increased in the same ratio over 11.26, as, when four cells were used or the magnetomotive effect doubled, it was increased above 6.53; and that the effect on the coil was now equal to 14.30. With regard to the difference in effect, when the coil was placed at equal distances from the poles, and when close to one of them, it may doubtless be obtained by a similar comparison. When one cell was in operation, it was 2.95, and, when the

magnetomotive power was doubled, 0.98; therefore, now that the power was increased by one fourth, the difference was probably 0.82 or very near it.

The bar was slid carefully out of the bow into the electro-magnet, and, on making a current, the galvanometer gave a deviation of 58.79, of which, according to the above, 44.49 may be considered as appertaining to the effect of the bar. The current was interrupted and the deviation following was 38.64.

In order to ascertain whether the large deviation obtained in the former series after the bar had been readjusted, was exclusively dependent on the fact, that the end-planes fitted closer to the iron-blocks of the electro-magnet, or whether it might also be owing to the bar having been moved lower down in the electromagnet, as there was reason to suppose, — the bar was moved very slightly, about 1mm, lower; and on opening the current, the next deviation, which, if no readjustment had taken place, would doubtless have been about 39, was now 46.56. From this we may conclude, what from other reasons had seemed probable, that the position of the points in the electromagnet, between which the bar is placed, may materially influence the extent of the molecular torsion.

The current was again made and broken four times, and gave a deviation always very near 39.72. After the current had passed for 3 hours, during which the bar had remained in the electro-magnet, it was interrupted, and gave a deviation of 40.31; and, on opening it again, 41.28. If the effect of the current on the coil, (14.30) be deducted from the average (40.08) of the above deviations, that appertaining solely to the bar will be 25.78, being the deviation corresponding to the molecular oscillations on making and breaking the current.

The distribution of the moments was then examined, whilst the bar remained in the electro-magnet under the influence of the current, and proved to be, after due allowance for the influence on the coil, such as shown by the curve *a a a*, Fig. 4, Pl. II. The position of this curve with regard to the scale, here again,

has no reference to the actual moments. After the current had been interrupted and the bar slid into the bow, the distribution of the moments was found to be that shown by the curve *b b b*.

The bar was now torn out of the bow, while the coil was fixed to the middle, causing a deviation equal to 57.45; after which the bar was replaced in the bow, and the two fixed in the electro-magnet. The current was allowed to pass for about $1/2$ an hour, after which it was broken, and the magnet and bow left in the electro-magnet through the night — about 12 hours. They were again taken out, and the differences of the moments were then as represented by the curve *c c c*. The coil was again fixed to the middle of the bar, and the bow being torn off, the deviation was 49.34; then the coil was quickly slid over the end, away from the bar, and the deviation was 24.32; after which the bar, with the coil replaced, was again fixed in the bow.

The last deviation, *e* in the figure, is consequently equal to the largest moment which could be retained in a free bar of such hardness. The sum of the two last deviations, or 73.66, was the moment at the middle of the bar before it was torn out of the bow the *last* time; and if it be supposed, as is probable, that the moment at the middle was the same when the bow had been torn off the previous evening, as now after being magnetized a second time with the same power, – namely 24.32 — then the moment, before the removal of the bow the *first* time, was equal to 57.45 + 24.32, or 81.77. How much it had been diminished by being removed into the bow, and by this being torn from the electro-magnet, could not be ascertained, but it must have been considerable.

The entire moment of the bar, while in the electro-magnet, might certainly have been found by first suddenly removing the bar with the coil on, from the electromagnet, and then by sliding the coil away — for the sum of the deviations caused by these two operations would represent the total moment. Such a proceeding, however, could not be employed, without interfering with the experiments by which were to be

ascertained the distribution of the moments *before* the endsurfaces of the bar had for any instant been deprived of contact with the bow. It must, on the whole, be a subject for an entirely new series of experiments, to ascertain the total moments at the different stages of magnetization.

As soon as the galvanometer had again come to rest, after the last mentioned experiment, the bar with the coil fixed to the middle was again inserted in the bow, and then torn out a second time (after the last magnetization), the deviation was then but 15.29
it was again inserted and torn out, deviation 9.09
— — — — 8.22
— — — — 8.00

The moment of the free magnet, in the middle, was now reduced to 17.32, at the north end it was 5.20, and at the south end 5.40.

After the bar had for 24 hours been compassed by the bow, no perceptible change of the moments had taken place. It then lay without the bow for fire hours, after which the moment was in the middle 16.98, at the north end 5.06 and at the south end 5.25. After it had been two days more without the bow, the respective moments were 16.50, 4.85 and 4.84; two days later 15.85, 4.53 and 4.62, — and the following day, when the bar had altogether been 120 hours with its ends free, the moments were respectively 14.95, 4.50 and 4.55, and the distribution in other respects as indicated by the curve *d d d*.

It will be observed from the figures, that a slight irregularity existed, with regard to the distribution of the moments, while the bar was in the bow, both while the current was in action and after it had ceased; but besides this accidental irregularity, — doubtless owing to unequal hardness of the steel — it will be seen, that the moments were always largest at the ends of the bar, while the current was in action, and smaller than at the middle, when it had ceased. In the first case, the relation between the bar and the polar-ends of the electro-magnet, is as that of an armature of soft iron to the polar-ends of a magnet, between which it is placed; the curves which represent the relation of

the moments are therefore, as well for the bar as for the iron-keeper, *concave*; thence it follows, that the free magnetism distributed on the side of the bar, is of the opposite kind to that of its nearest polar plane, — that consequently North-magnetism is distributed on the side of the bar in an increasing ratio towards that end-surface, which itself is South-magnetic and vice versa — South-magnetism towards the North-magnetic end-plane.

The reason why the moments become smaller near the middle of the bar, is clearly owing to the resistance encountered by the magnetizing force, — or to the resistance against the molecular torsion offered by the entire bar.

When the current is interrupted, the relations mentioned are reversed; the moments then become smallest near the ends, and greatest at the centre of the bar; the free North-magnetism on the side is exchanged for South-magnetism, and the South- for North-magnetism, so that the magnetism on the side and nearest end-plane come to be of the same kind.

As in the former case, it was the resistance against molecular torsion in the steel bar, which caused the line corresponding to the distribution of the moments to be not straight, but concave; so now it is the presence of the same force in the bow, which causes the line to be *convex*. It is only resistingly and in order to attain mutual saturation, that the molecules of the bow or keeper allow themselves to be deflected, and that those of the magnet allow the deflection they have received by magnetization, to be diminished; they accomodate themselves, however, to each other, till the tension, caused by the receding motion within the limit of elasticity in the magnet, has attained a magnitude in equilibrium with the resistance to deflection offered by the molecules of the bow or keeper.

The fact that the distribution of the moments was not alike after both the last magnetizations (pp. 42), Fig. 4. *b b b* and *c c c*, seems to prove, that the effect of the diminution of the molecular deflection beyond the limit of elasticity, which took place the instant that the bow was torn from the bar, after the first

magnetization, had not wholly been eliminated by its being remagnetized by the same force as before.

In the case of an experiment which was at last made, of magnetization by means of a weak current from only one Meidinger-cell, it was considered superfluous to make any measurement of the differences between the moments while the bar remained in the electro-magnet or in the bow, — and the observations were limited therefore principally to measuring the deviations by the opening and breaking of the current, which showed results somewhat different from those obtained in the previous experiments.

The same steel-bar as before was used, and after it had been made redhot and rehardened, placed in the electro-magnet, and the induction-coil was attached to the middle. Upon starting the electric current, the galvanometer, through which the undiminished inductive-current was now passed, gave a deviation 3.17
current interrupted 2.05
„ opened 2.47
„ interrupted 2.18
„ opened 2.38
„ interrupted 2.06

It will be noticed, that the first deviation, though larger than those following, is yet in comparison with them, smaller than the first deviation obtained in the experiments with stronger currents, is to the succeeding ones representing the oscillations of the molecules within the limit of elasticity. It will also be observed, that the deviations were always larger on the making than on the breaking of the current, which was not the case, or at least could not be noticed, when the stronger currents were used and only a small portion of the induced current passed through the galvanometer. During the weak magnetization, when the angle formed by the molecular axes in their normal position and in the position at the limit of their elasticity, is very small, and consequently the resistance they have to surmount, in the latter position, against

permanent deflection, is very slight, it seems that the effect of the repeated magnetizing shocks on opening the current, enables them, to some small extent, to overcome the friction and to produce a permanent deflection, and that this goes on till the resistance, in consequence of the increased torsion, prevents its continuance.

III.

The dynamic influence which magnetic bodies exercise on each other, depends not only on the amount of free magnetism in each, but also on* the mode in which it is distributed, on the shape of the bodies and on their mutual distances; — the rational determination of the attraction between such bodies, therefore presupposes a knowledge of these relations.

Coulomb, by the aid of his torsion-balance, investigated the magnetic distribution over the side of a free magnet i. e. a magnet to which there is no keeper attached. The chief features of this mode of investigation were the following: A magnetic needle, suspended by a wire and moving in the horizontal plane, placed itself without torsion of the wire in the magnetic meridian. Along the side of the magnet under investigation was fixed a piece of wood of uniform thickness. The magnet was then placed in a vertical position so as to allow the strip of wood to touch the side of the needle at one end, and was so arranged that it could be moved up or down, and every point along the side be placed right opposite the needle. The attraction taking place at these different points, at a distance equal to the width of the intervening strip of wood, was ascertained by the torsion of the wire-twisting until the needle was forced away from the piece of wood. As the attraction under these circumstances was in direct proportion to the product of the free magnetism in the needle and that in the part of the magnet immediately opposite, *Coulomb* concluded that as the same part

of the needle always touched the strip of wood, these products would be proportional to the free magnetism at the different points in the magnet where the attraction had been tested.

Coulomb also applied another method which led to a result very nearly the same. A magnetic needle oscillating freely in the horizontal plane was supported opposite a vertically placed magnet which could be moved up and down, so that any part might be brought opposite the needle at a constant distance as before. From the number of vibrations made by the needle in a given time according to the position of the magnet, he calculated the free magnetism at different points on the side.

Biot subsequently made an analysis of the results thus obtained by *Coulomb*, and was thereby led to conclude, that the free magnetism along the side of a magnet is proportional to the ordinates in a catenary, the distance between the suspension-points of which is the same as the length of the magnet. *Van Rees*, who more recently treated this subject, has shown that the conclusion arrived at by *Biot*, as well as those arrived at by *Lenz* and *Jacobi*, that the molecular moments along the sides of electro-magnets vary as the ordinates of a parabola, are to be regarded as merely approximations to the truth.

It will probably be clear from what will subsequently appear, that *Coulomb's* mode of investigation could not possibly lead to a reliable result, since while the magnet and needle are approaching, a local disturbance arises in the magnetic distribution, both in the magnet, occasioned by the needle, and in the needle, occasioned by the magnet; so that the quantity of free magnetism collected at the end of the needle varies according to the part of the magnet to which it is opposite, and the free magnetism in the various parts of the magnet is of different intensity before and after the approach of the needle: in other words, the distribution of the free magnetism of the bar is ascertained by a process which not only disturbs that distribution, but which also destroys the supposed constancy of the force in the needle by which it is to be measured.

Having satisfied myself on this point, I naturally sought for a method by which a perfectly reliable measurement of the magnetic distribution, under varying circumstances, might be obtained; and this I found in the system adopted by *van Rees*, the principle of which is the same as that which *Lenz* and *Jacobi* employed for measuring the moments of electro-magnets.

According to the method of *van Rees*, magnetic moments are determined by the electric currents which they induce, and from the distribution of the moments the free magnetism may be ascertained. This method has been employed both in the general investigations on the relations of moments to one another, described above, and in the more detailed experiments which will follow. It is one that has been repeatedly adopted of late years for examinations of a similar kind, and is based upon a theory expounded by *van Rees* in *Poggendorff's* Annalen Vol. LXX. The relation between the moments is determined by the electromotive force induced, when the current produced in the coil is passed through a mirror-galvanometer; it is in fact proportional to the *sines* of half the angles as read off in the mirror.

Although the electromotive force of the current only depends on the molecular moments that produce it, and is uninfluenced by any other force, the practical measurement according to the principles just mentioned, is, nevertheless not entirely free from error.

In order that the electromotive force of the current might represent exactly the moment of a given transverse plane in the magnet, the length of the coil would have to be infinitely small; but as this is impossible, the current which passes does not represent the moment in any single plane, but in the whole part of the magnet covered. However much the moments in the magnet may vary in general, the variation in the small portion covered by the coil may be considered as in an arithmetical proportion to the length, so that the moment at the centre of the coil is equal to the average of the moments in the whole portion covered. If the helix were acted upon solely by this portion, the deviation of

the galvanometer might be considered as representing the moments in the plane passing through the middle of the length of the coil. But the moments of the parts on the side of the coil also act upon it, although in a proportion decreasing with the distance. When situated anywhere on the magnet, not very near either pole-end, it may be gathered that the somewhat stronger influence on one side is balanced by the less strong on the other, and that the deviation of the mirror is therefore proportional to the moments in the transverse plane of the magnet which passes through the middle of the coil. If, however, the coil be situated on the magnet at its end or near to it, it is acted upon only by one side, and the force will therefore be slightly weaker than it should be in order to correspond with the moments in the plane through the middle of the coil; this has also been pointed out by *Rothlauf* in *Poggendorff's* Annalen Vol. CXVI p. 592. For the measurements to which I shall presently refer, I have employed cylindrical magnets and keepers, the dimensions of which are all 146^{mm} in length and 10.5^{mm} in diameter, except in certain cases; the coil for the induced current fits close to the side of the magnet and keeper, and is 10.9^{mm} long, with a wire helix 7.5^{mm} deep, the gauge of the wire being 0.7^{mm}. This wire was joined to a mirror-galvanometer whose centimetre scale, with the telescope, was placed 2 meters from the galvanometer. It has been mentioned before that the electromotive force of the induced currents is proportional to the *sine* of $^1/_2$ the angles of deviation read off the scale of the galvanometer. As these angles were not large, their arcs nearly coincided with the sines and tangents, and I considered it unnecessary to calculate the angles themselves; the reading on the galvanometer scale, equal to the tangent of twice the angle of deviation, has therefore been taken unreduced in the series of observations below.

The mode of measurement as regards the free magnet (i. e. one to which no keeper is attached) has been carried out in such a way, that after the coil had been placed on the magnet at a fixed distance from one of the pole-ends, and the oscillation occasioned by it in the galvanometer had ceased,

the coil was rapidly drawn away over the nearest end to some distance from the magnet, and at the same time turned so as to place its plane at right angles to the magnetic axis. The deviations thus occasioned were read off the mirror. For other distances from the polar ends the same plan was adopted.

When the moments of the magnet were measured while under the influence of a keeper the position of the latter prevented the coil from being drawn over the end facing it; and likewise when the moments of the portion nearest the keeper were to be determined, it was difficult to place the magnet so that the coil could be drawn the other way over the middle and further end; the coil was therefore left in its place on the magnet, and the keeper quickly removed. The deviation obtained by thus annulling the induction caused by the keeper, will, when added to the deviation for the same transverse plane previous to the application of the keeper, give the total molecular moment of that part of the magnet while under the influence of the keeper. In measuring the moments of the keeper, it was left in the coil, and the magnet quickly withdrawn. The current was in this case produced by annulling the induction caused by the magnet.

The distance from the end to the first transverse plane examined in the magnet, was equal to half the length (5.45^{mm}) of the induction coil, so that the plane of the coil and the surface of the end of the magnet coincided with one another while the first measurement was taken; the distance from the end of the magnet to the next transverse plane for which the moment was determined, was 15^{mm}, and the consecutive intervals were 15^{mm}.

The moments in the keeper were also measured at a distance from each end equal to half the length of the coil, and the consecutive measurements on the side near the magnet followed each other at distances shorter than at the further end.

Two parallel series of observations were made with magnets of the same dimensions but of different intensity, in order to ascertain how far intensity might influence the distribution of the moments. The tables below comprise the moments of a magnet when beyond the influence of a keeper, when in contact with it, and when apart from it at twelve different distances.

Deviations for molecular moments.

Magnet of great intensity.

Without keeper.

Distance fr. N-end	Deviation	Distance fr. S-end	Deviation
5.5mm	7.02	5.5mm	6.71
15	10.83	15	10.18
30	15.20	30	14.64
45	17.52	45	17.26
60	19.04	60	18.82
		75	19.42

With keeper in contact.

Magnet		Keeper	
Distance	Deviation	Dist. fr. end oppos. Magn.	Deviation
5.5mm fr. N End	15.97	5.5mm	14.56
15	17.63	15	13.40
30	19.54	25	12.35
45	20.70	35	11.26
60	21.41	50	9.67
75mm fr. S End	21.22	65	8.22
60	20.06	80	6.78
45	18.22	95	5.40
30	15.23	110	4.02
15	10.58	125	2.75
5.5	6.87	140.5	1.57

The keeper removed 0.284mm

5.5mm fr. N End	14.04	5.5mm	11.81
15	16.18	15	10.97
30	18.62	25	10.05
45	20.07	35	9.29
60	20.89	50	8.03
75mm fr. S End	20.82	65	6.90
60	19.82	80	5.78
45	17.93	95	4.69
3	15.10	110	3.58
15	10.49	125	2.50
5.5	6.83	140.5	1.42

The keeper removed 0.795^{mm}.

Magnet		Keeper	
Distance	Deviation	Dist. fr. end oppos. Magn	Deviation
5.5^{mm} fr. *N* End	12.08	5.5^{mm}	9.64
15	14.80	15	9.28
30	17.73	25	8.72
45	19.30	35	8.03
60	20.35	50	7.04
75^{mm} fr. *S* End	20.51	65	6.08
60	19.60	80	5.08
45	17.75	95	4.09
30	14.98	110	3.12
15	10.41	125	2.18
5.5	6.80	140.5	1.28

The keeper removed 1.690^{mm}

5.5 fr. *N* End	9.94	5.5^{mm}	7.36
15	13.25	15	7.13
30	16.83	25	6.82
45	18.68	35	6.40
60	19.90	50	5.59
75 fr. *S* End	20.08	65	4.92
60	19.31	80	4.07
45	17.59	95	3.26
30	14.87	110	2.49
15	10.33	125	1.70
5.5	6.77	140	1.05

The keeper removed 3.375^{mm}

5.5^{mm} fr. *N* End	8.86	5.5^{mm}	5.11
15	12.47	15	5.09
30	16.27	25	4.94
45	18.33	35	4.65
60	19.62	50	4.11
75^{mm} fr. *S* End	19.90	65	3.58
60	19.13	80	3.00
45	17.47	95	2.43
30	14.78	110	1.88
15	10.27	125	1.36
5.5	6.75	140.5	0.88

The keeper removed 5.110mm.

Magnet		Keeper	
Distance	Deviation	Dist. fr end oppos. Magn	Deviation
5.5mm fr. *N* End	8.00	5.5mm	4.43
15	11.70	15	4.45
30	15.71	25	4.33
45	17 97	35	4.10
60	19.39	50	3.60
75mm fr. *S* Eud	19.72	65	3.14
60	19.00	80	2.63
45	17.36	95	2.12
30	14.72	110	1.69
15	10.23	125	1.22
5.5	6.73	140	0.85

The keeper removed 7.620mm

5.5mm fr. *N* End	7.61	5.5mm	3.20
15	11.35	15	3.33
30	15.54	25	3.35
45	17.80	35	3.22
60	19.23	50	2.90
75 fr. *S* End	19.62	65	2.57
60	18.93	80	2.18
45	17.32	95	1.80
30	14.69	110	1.42
15	10.21	125	1.00
5.5	6.72	140.5	0.80

The keeper removed 10.96mm

5.5mm fr. *N* End	7.34	5.5mm	2.46
15	11.08	15	2.60
30	15.40	25	2.63
45	17.67	35	2.60
60	19.13	50	2.42
75 fr. *S* End	19.53	65	2.18
60	18.88	80	1.90
45	17.29	95	1.58
30	14.67	110	1.22
15	10.20	125	0.94
5.5	6.72	140.5	0.72

The keeper romoved 18.24mm.

Magnet		Keeper	
Distance	Deviation	Dist. fr. end oppos. Magn	Deviation
5.5mm fr. N End	7.18	5.5mm	1.75
15	10.96	15	1.88
30	15.30	25	1.94
45	17.60	35	1.94
60	19.08	50	1.80
75 fr. S End	19.48	65	1.64
60	18.85	80	1.46
45	17.28	95	1.20
30	14.66	110	1.00
15	10.19	125	0.77
5.5	6.72	140.5	0.58

The keeper removed 29.39mm

5.5mm fr. N End	7.10	5.5mm	1.18
15	10.90	15	1.35
30	15.25	25	1.41
45	17.56	35	1.41
60	19.06	50	1.34
75mm fr. S End	19.45	65	1.20
60	18.84	80	1.07
45	17.27	95	0.95
30	14.65	110	0.80
15	10.19	125	0.63
5.5	6.72	140.5	0.49

The keeper removed 39.21mm

5.5mm fr. N End	7.07	5.5mm	0.90
15	10.87	15	1.02
30	15.23	25	1.12
45	17.54	35	1.15
60	19.05	50	1.09
75 fr. S End	19.44	65	0.99
60	18.83	80	0.88
45	17.27	95	0.77
30	14.65	110	0.66
15	10.18	125	0.55
5.5	6.71	140.5	0.40

The keeper removed 55.54^{mm}.

Magnet		Keeper	
Distance	Deviation	Dist. fr. end oppos. Magn	Deviation
5.5^{mm} fr. *N* End	7.04	5.5^{mm}	0.65
15	10.85	15	0.75
30	15.22	25	0.87
45	17.53	35	0.89
60	19.05	50	0.87
75 fr. *S* End	19.43	65	0.74
60	18.83	80	0.67
45	17.26	95	0.60
30	14.64	110	0.52
15	10.18	125	0.45
5.5	6.71	140.5	0.36

The keeper removed 78.42^{mm}

5.5^{mm} fr. *N* End	7.03	5.5^{mm}	0.41
15	10.84	15	0.52
30	15.21	25	0.57
45	17.53	35	0.61
60	19.05	50	0.59
75 fr. *S* End	19.43	65	0.55
60	18.83	80	0.50
45	17.26	95	0.45
30	14.64	110	0.40
15	10.18	125	0.34
5.5	6.71	140.5	0.28

Magnet of weaker intensity.

Without keeper.

Distance fr. *N* End	Deviation	Distance fr. *S* End	Deviation
5.5^{mm}	4.30	5.5^{mm}	3.58
15	6.85	15	5.72
30	9.83	30	8.12
45	11.87	45	9.86
60	12.49	60	11.20
		75	12.14

With keeper in contact.

Magnet		Keeper	
Distance	Deviation	Dist. fr. end oppos. Magn	Deviation
5.5^{mm} fr. *N* End	10.14	5.5^{mm}	9.31
15	11.20	15	8.67
30	12.64	25	8.02
45	13.57	35	7.37
60	13.60	50	6.34
75 fr. *S* End	13.11	65	5.32
60	11.97	80	4.41
45	10.48	95	3.50
30	8.54	110	2.56
15	6.01	125	1.71
5.5	3.75	140.5	0.89

The keeper removed 0.284^{mm}

5.5^{mm} fr. *N* End	9.90	5.5^{mm}	7.78
15	10.24	15	7.24
30	12.04	25	6.69
45	13.16	35	6.14
60	13.35	50	5.28
75^{mm} fr. *S* End	12.88	65	4.44
60	11.79	80	3.68
45	10.34	95	2.90
30	8.44	110	2.11
15	5.94	125	1.41
5.5	3.71	140.6	0.76

The keeper removed 0.795mm.

Magnet		Keeper	
Distance	Deviation	Dist. fr. end oppos. Magn	Deviation
5.5mm fr. *N* End	7.53	5.5mm	6.16
15	9.34	15	5.71
30	11.56	25	5.30
45	12.85	35	4.87
60	13.09	50	4.22
75mm fr. *S* End	12.69	65	3.55
60	12.65	80	2.93
45	10.22	95	2.28
30	8.36	110	1.70
15	5.89	125	1.18
5.5	3.68	140.5	0.64

The keeper removed 1.690mm

5.5mm	6.31	5.5mm	4.77
15	8.46	15	4.57
30	11.03	25	4.33
45	12.54	35	4.00
60	12.89	50	3.49
75mm fr. *S* End	12.51	65	2.96
60	11.53	80	2.43
45	10.12	95	1.89
30	8.29	110	1.42
15	5.84	125	0.99
5.5	3.65	140.5	0.56

The keeper removed 3.375mm

5.5mm fr. *N* End	5.50	5.5mm	3.51
15	7.83	15	3.50
30	10.57	25	3.38
45	12.33	35	3.17
60	12.76	50	2.76
75mm fr. *S* End	12.38	65	2.36
60	11.43	80	1.94
45	10.04	95	1.56
30	8.24	110	1.19
15	5.80	125	0.80
5.5	3.61	140.5	0.50

The keeper removed 5.110^{mm}.

Magnet		Keeper	
Distance	Deviation	Dist. fr. end oppos. Magn	Deviation
5.5^{mm} fr. *N* End	5.03	5.5^{mm}	2.77
15	7.45	15	2.86
30	10.32	25	2.83
45	12.17	35	2.69
60	12.67	50	2.36
75^{mm} fr. *S* End	12.30	65	2.02
60	11.34	80	1.68
45	9.97	95	1.35
30	8.20	110	1.00
15	5.77	125	0.68
5.5	3.59	140.5	0.43

The keeper removed 7.620^{mm}

5.5^{mm} fr. *N* End	4.79	5.5^{mm}	2.13
15	7.25	15	2.21
30	10.16	25	2.23
45	12.07	35	2.16
60	12.61	50	1.91
75^{mm} fr. *S* End	12.25	65	1.64
60	11.29	80	1.37
45	9.93	95	1.09
30	8.17	110	0.85
15	5.75	125	0.60
5.5	3.59	140.5	0.36

The keeper removed 10.96^{mm}

5.5^{mm} fr. *N* End	4.60	5.5^{mm}	1.60
15	7.13	15	1.75
30	10.08	25	1.78
45	12.02	35	1.75
60	12.58	50	1.57
75 fr. *S* End	12.23	65	1.38
60	11.27	80	1.16
45	9.91	95	0.93
30	8.16	110	0.71
15	5.74	125	0.50
5.5	3.59	140.5	0.30

The keeper removed 18.24^{mm}.

Magnet		Keeper	
Distance	Deviation	Dist. fr. end oppos. Magn	Deviation
5.5^{mm} fr. *N* End	4.47	5.5^{mm}	1.15
15	6.99	15	1.30
30	9.95	25	1.30
45	12.95	35	1.27
60	12.54	50	1.16
75^{mm} fr. *S* End	12.18	65	1.03
60	11.23	80	0.87
45	9.89	95	0.72
30	8.14	110	0.56
15	5.73	125	0.41
5.5	3.59	140.5	0.25

The keeper removed 29.39^{mm}

5.5^{mm} fr. *N* End	4.39	5.5^{mm}	0.72
15	6.92	15	0.80
30	9.89	25	0.89
45	12.91	35	0.90
60	12.52	50	0.83
75^{mm} fr. *S* End	12.16	65	0.80
60	11.22	80	0.69
45	9.87	95	0.57
30	8.13	110	0.45
15	5.73	-125	0.34
5.5	3.59	140.5	0.21

The keeper removed 39.11^{mm}

5.5^{mm} fr. *N* End	4.35	5.5^{mm}	0.64
15	6.89	15	0.69
30	9.86	25	0.71
45	12.89	35	0.71
60	12.51	50	0.70
75 fr. *S* End	12.15	65	0.62
60	11.22	80	0.54
45	9.86	95	0.46
30	8.13	110	0.35
15	5.73	125	0.24
5.5	3.59	140.5	0.17

The keeper removed 55.54mm.

Magnet		Keeper	
Distance	Deviation	Dist. fr. end oppos. Magn	Deviation
5.5mm fr. N End	4.32	5.5mm	0.42
15	6.87	15	0.48
30	9.84	25	0.53
45	12.88	35	0.51
60	12.51	50	0.47
75mm fr. S End	12.15	65	0.42
60	11.22	80	0.39
45	9.86	95	0.33
30	8.13	110	0.25
15	5.72	125	0.17
5.5	3.58	140.5	0.13

The keeper removed 79.42mm

5.5mm fr. N End	4.31	5.5mm	0.29
15	6.86	15	0.32
30	9.82	25	0.36
45	11.86	35	0.40
60	12.50	50	0.38
75 fr. S End	12.14	65	0.34
60	11.20	80	0.28
45	9.86	95	0.22
30	8.12	110	0.16
15	5.72	125	0.12
5.5	3.58	140.5	0.09

By means of these two systems, each consisting of fourteen series of observations, an equal number of curves may be drawn, showing the relative magnitudes of the moments both in magnet and keeper along their axes, while under each other's influence. Plate III, fig. 1, and Plate IV are such curve-systems in which the abscissæ represent the distances from the ends to the respective transverse sections and the ordinates the corresponding moments.

Since the free magnetism is proportional to the difference between, or to the differential quotients of the moments in the consecutive sections, other curves may be constructed from the curves of moments, whose ordinates are proportional to these

differences or differential quotients, representing the distributi of the free magnetism over the sides. Such curves, correspondi to the curves of moments, Plate III, fig. 1, are represented Plate III, fig. 2. To obviate indistinctness arising from the coı plication of the system belonging to the keeper, the ordinat are given double the length of those for the magnet. Thus, dra to the same scale as the curves *a c e*, *a d f*, *&c*, that for t keeper in contact with the magnet should be represented by t dotted curve *h i*.

The amount of free magnetism distributed over the surfa of the ends, which, at least at short distances between magı and keeper plays the most prominent part in the phenomena attraction, cannot be directly inferred from the moments.

An attempt was made by the method indicated by *Schneebe* (Programm der eidgen. polytechnischen Schule, Zürich 1871) determine the polar distance from the ends, — and from this, in co nection with the distribution of the side-magnetism, to arrive the relative quantity of magnetism spread over the ends.

Schneebelli calculates the polar distance by the form

$$2\,\lambda^2 = \frac{r_{,}^{3}\ \text{tang}\ \varphi - r^3\ \text{tang}\ \varphi}{r^5\ \text{tang}\ \varphi - r_{,}^{5}\ \text{tang}\ \varphi_{,}}\ r_{,}^{2}\ r^2$$

in which λ represents half the distance between the magn poles, r the distance between the centre of the magnet and moveable needle, and φ the angle of deviation.

The practical application of this formula, however, proved ficult, as the expressions r^3 tang φ and r^5 tang φ varied v slightly with the distances at which the measurements took pla For this reason the numerator and denominator became so sn that the unavoidable errors of observation affected the value (in a very great degree, and it was necessary to determine t value exactly if the object in view was to be obtained. I soug therefore, other and better means to determine the relat between the quantity of magnetism on the side and end the bar.

It will be seen from Plate III, fig. 2, that the free magnet

extended over the left half of the side of the magnet is slightly increased when a keeper is brought in contact with the end-plane of the other half, and that this augmentation is greatest near the centre, while it decreases towards the end, where it gradually vanishes. The area of fig. *a b c* represents the free magnetism in the left half when the magnet is not acted upon by a keeper, and *a b d* the magnetism when a keeper is brought in contact with the right-hand end of the magnet. Therefore, as the augmentation gradually disappears on the side towards the left end, it seems probable that on the end-plane itself there is no augmentation whatever, or that it must in any case be very slight. This also seemed verified by experiment.

A needle, 70^{mm} long, was caused to oscillate in front of the end-plane, say negative, of a magnet A at a distance of 40^{mm}, and the number of vibrations per minute was counted, after which the negative pole-end of a magnet B, almost equally strong was brought into contact with the positive end of the first mentioned magnet A, and the vibrations were again counted.

The magnet B now caused the same effect as a keeper, but in a much greater degree. The free magnetism in the half of A farthest from the needle was for the most part saturated, and therefore had scarcely any influence in diminishing the effect of the other pole on the needle; the number of oscillations should therefore increase. The increase of free magnetism on the left half of A, as shown in fig. 2, *a c d*, also contributed slightly towards increasing the number of vibrations. The weakening influence exerted on the needle by the remote end of B can be left out of consideration on account of its great distance. The needle was first placed at an angle of deviation of 5^0, and made on an average 37.7 vibrations per $\frac{1}{2}$ minute previous to the approach of B, and 39.5 afterwards. This change, corresponding to an increase of force of about 10 per cent, seems to be no more than what might have been expected from the alteration in the magnetic condition above mentioned, and hence, if the acceleration was at all owing to any increase of free magnetism in the *end-plane*, that increase

must have been very slight. The number of vibrations would have been still less increased if an iron keeper, instead of a magnet, had been brought in contact with A; the increase, — if any, — of the magnetism at the end-surface may under all circumstances be considered as infinitely small compared with the total magnetism, and with perfect safety be left out of account altogether. The free magnetism in the end-plane of the magnet, remote from the keeper, will therefore in future be taken as constant.

The quantity of North-magnetism existing in a magnetic body corresponds of course to an equal quantity of South-magnetism, either free or in a state of saturation, when affected by another magnetic body. When one end of a keeper is in contact with a magnet, *van Rees* has shown that the magnetism on the whole exposed or free surface of the keeper is of the same kind as that of the magnet-end nearest to it. The plane in contact with the magnet must therefore contain a quantity of magnetism, equal in amount and of opposite kind to that spread over the exposed surface. And this quantity, saturating as it does the magnetism of the magnet's end-surface, must of course be equal to it; the sum of this and that still left in the half of the magnet nearest the keeper, equals the total quantity of free magnetism in the other half of the magnet. This latter half will in future be considered as negative. The quantity of free magnetism on the sides of the magnet and keeper can be calculated from the measurements (pp. 52—61); and if it were possible to ascertain the small quantity spread over the plane at the positive end of the keeper, we could determine the total amount in the whole bar. As the free side-magnetism on the magnet and keeper, when in contact, is represented by curves whose ordinates are proportional to the differences between the moments, the areas enclosed between the curves and the base line, fig. 2, Plate III, may be considered as represented by the differences between the largest ordinate and those at the ends of the curves of moments fig. 1, or by the vertical projection of this curve; so that the free side-magnetism

(area $a\ b\ d$, fig. 2) on one side of the point of indifference, a, fig. 1, is represented by $a\ b$ minus $c\ d$, and that of the other side (area $d\ f\ g + g\ h\ i\ k$, fig. 2) by $a\ b$ minus $e\ f$, fig. 1. Now as the free side-magnetism plus the free end-magnetism gives the same result on both sides, the free end-magnetism may be represented either by the ordinates $c\ d$ and $e\ f$ alone or by these amounts plus or minus a constant. For the same reason, when the magnet is beyond the influence of a keeper, the ordinate $g\ h$, fig. 1, indicates the positive free end-magnetism, if $c\ d$, which is somewhat smaller, corresponds to the negative end-magnetism. We may here notice a fact, shown by the series of moments (pp. 52—56), that the magnetism is not equally distributed over the two halves of the magnet, even when not acted upon by any other magnetic body; nor is the point of indifference situated exactly in the centre of the magnet. This inequality exists probably more or less in all magnets, but it does so in a strikingly high degree in the weaker magnet B, of which the molecular moments have been examined. (pp. 57—61, Plate IV).

We will now deal more fully with the relation between the free magnetism on the side- and end-surfaces.

The limiting ordinates $c\ d$ ane $e\ f$, fig. 1, represent the magnetic moments or molecular deflection of the extreme transverse strata of the magnet and keeper. The magnetism of the surface-molecular-poles is not here saturated by the opposite poles of any neighbouring stratum, and appears therefore as free magnetism, in amount equal to their intensity multiplied by the *sine* of their angle of torsion. But the molecular moments and consequently the lengths of the ordinates are likewise proportional to this *sine*, so that the limiting ordinate is proportional both to the molecular moments of the end-stratum and to the free magnetism of the end-plane.

The case may also be considered from another point of view. The free unsaturated magnetism in any transverse stratum is proportional to the difference between the moments in this stratum and those in the one next to it. Now, as there is no stratum beyond the extreme one, the moments there are equal to

zero, the difference, consequently, is simply equal to the moments of the extreme stratum, and the free magnetism is therefore proportional to it.

From the ends to the point of indifference, situated near the centre of the magnet, the torsional angle of the molecules increase, and hence the magnetism of the strata lying behind one another is not entirely saturated. The unsaturated quantities, proportional to the differences between the *sines* of the neighbouring angles of torsion, appear as free magnetism distributed over the side; and, as before shown, the collected quantity of this magnetism is proportional to the difference between the longest ordinate near the centre and those at the ends. The longest ordinate, proportional to the largest molecular moment, is therefore also proportional to the sum of the free magnetism on either side of the point of indifference added to that on the end-plane. The case is the same with the keeper. The limiting ordinates are proportional both to the molecular moments in the end strata and to the free magnetism in the same. If the longest ordinate does *not* coincide with the limiting ordinate near the magnet, it is yet, as in the case of the magnet, proportional to the sum of the free magnetism distributed over the lateral and terminal surfaces on either side of the ordinate in question. If, it coincides with the limiting ordinate, only one kind of magnetism is distributed over the whole lateral surface.

The ratio between the free magnetism on the side- and end-surfaces may consequently be immediately determined in the simplest manner by means of the molecular moments.

The relative quantities have been determined at those 14 distances, between 0 and ∞, for which the moments were measured as above in the two magnets and keepers of unequal strength. The relations are given in the following tables, in which the total free magnetism, when the magnet is in contact with the keeper, is taken as 1000.

Magnet with greatest intensity.

Distance between magn. & keeper	Magnet				Keeper			
	Negative end-plane	Negative side-portion	Positive side-portion	Positive end-plane	Negative end-plane	Negative side-portion	Positive side-portion	Positive end-plane
0	— 191.3	— 808.7	+ 294.0	+ 706.0	— 706.0	0	+ 653.3	+ 52.7
0.284mm	„	789.5	385.8	595.0	576.5	0	528.2	48.3
0.795	„	768.3	478.6	481.0	456.3	0	412.9	43.4
1.690	„	749.7	564.4	376.6	346.8	0	308.2	38.6
3.373	„	739.4	634.5	296.2	237.0	0	204.1	32.9
5.110	„	730.5	658.2	263.6	201.0	— 10.4	180.3	31.1
7.620	„	727.2	675.8	242.7	141.3	16.9	130.7	27.5
10.96	„	723.9	687.0	228.2	110.0	17.9	102.4	25.5
18.24	„	722.4	698.8	214.9	75.5	17.7	70.7	22.5
29.39	„	721.5	704.7	208.1	49.9	17.4	47.9	19.4
39.21	„	720.9	706.4	205.8	36.1	16.5	35.8	16.8
55.54	„	720.5	707.9	203.9	26.5	15.5	28.2	13.8
78.42	„	720.1	708.9	202.5	14.8	13.6	16.7	11.7
∞		720.1	709.4	202.0	0.0	0.0	0.0	0.0

Magnet with least intensity

0	— 152.1	- 847.9	+ 293.2	+ 706.8	— 706.8	0	+ 658.6	+ 48.2
0.284mm	„	827.4	383.7	595.8	585.1	0	544.2	40.9
0.795	„	809.1	481.1	480.1	458.0	0	423.6	34.4
1.690	„	791.9	570.0	374.0	348.0	0	318.0	30.0
3.373	„	781.4	639.0	294.5	250.4	0	224.1	26.3
5.110	„	774.5	663.5	263.1	199.5	— 11.0	187.8	22.7
7.620	„	771.1	681.3	241.9	147.1	16.5	144.6	19.0
10.96	„	768.3	693 2	227.2	113.6	17.0	114.5	16.1
18.24	„	766.0	704.7	213.4	77.5	17.5	81.8	13.2
29.39	„	764.5	710.5	206.1	49.5	16.2	54.7	11.0
39.21	„	763.7	714.1	201.7	36.5	15.3	43.0	8.8
55.54	„	763.1	715.2	200.0	24.0	14.7	32.1	6.6
78.42	„	762.7	715.5	199.3	15.8	13.4	24.1	5.1
∞	„	762.2	715.5	198.8	0.0	0.0	0.0	0.0

A first glance at these tables might lead to the supposition, that no very great harmony exists between the distribution of magnetism in the two systems. This incongruity is, however, to a great extent only apparent, and depends, as regards the magnets, chiefly on that absence of symmetry in the distribution of the moments on each side of the point of indifference, which exists

in both, but to a peculiar extent in the weaker magnet. In the latter magnet, the point of indifference lies about $^2/_5$ths of the length of the magnet from the positive, and $^3/_5$ths from the negative end. In consequence of this, the equal quantities of magnetism on each side of the point of indifference are distributed over the side and end-planes in a somewhat different proportion. If the numbers representing the *total* magnetism in the two systems are compared, both in magnet and keeper, at different distances from one another, the harmony is very great and shows, that the intensity of the magnet does not affect the relative distribution of the quantity of magnetism.

The total magnetism in magnet and keeper.

Distance in millimeters	0.0	0.284	0.795	1.690	3.373	5.110	7.620
In magn. with greater intens.	1000.0	980.8	959.6	941.0	930.7	921.8	918.5
„ „ „ smaller „	1000.0	979.5	961.2	944.0	933.5	926.6	923.2
„ Keeper to stronger magnet	706.0	576.5	456.3	346.8	237.0	211.4	158.2
„ to weaker „	606.8	585.1	458.0	348.0	250.4	210.5	163.6
Distance in millimeters	**10.96**	**18.24**	**29.39**	**39.21**	**55.54**	**78.42**	**∞**
In magn. with greater intens.	915.2	913.7	912.8	912.2	911.8	911.4	911.4
„ „ „ smaller „	920.4	918.1	916.6	915.8	915.2	914.8	914.3
„ Keeper to stronger magnet	127.9	93.2	67.3	52.6	42.0	28.4	0.0
„ to weaker „	130.6	95.0	65.7	51.8	38.7	29.2	0·0

In these series, the incongruities amount only to a fraction per cent., as far as the magnets are concerned; whereas those for the keeper in some cases rise to several per cent. But these inconsistencies vary sufficiently to show that they are chiefly casual;

and are easily explained; both by the measurement having as a rule been performed only once, and because, although I had a separate room in the University for my operations, still the working of the students in the neighbouring rooms affected my instrument so perceptibly, that in one or two cases I found it necessary to cease working altogether. It must also be borne in mind that the deviations appertaining to the keeper, especially under the influence of the weaker magnet, were partly very small, making the errors of observation of greater importance. Moreover, there may really exist a difference, although small, in the distribution of the moments in the *magnet*, on account of that different degree of their permanent diminution, which the parts near the ends have suffered.

With regard to the distribution of the magnetism in the *keeper* on each side of the point of indifference — where such a point exists — there is, when the induced magnetism is very weak, a considerable difference in the two series, which, however, does not depend on the strength of the inducing magnet. When the deflecting force is small, the resistance that the molecules offer to motion becomes very apparent, and in order to obtain uniform deviation for moments in two series of observations, principally in any transverse plane between the point of indifference and the end farthest from the magnet, it is necessary to allow some time to pass before removing the magnet from its position in front of the keeper; and, by gently tapping the keeper, to assist the molecules in taking up their new positions. I only discovered this after having made the greater part of the investigations, while endeavouring to ascertain the reason of the irregularities, which often and unexpectedly presented themselves when the deviating forces were small. Though I am not fully satisfied on the point — not having had time to go more closely into the matter, — it appeared, as if the molecules moved with greater ease under the influence of weak induction, when they have been powerfully deflected for some time in both directions, than after they have been long in a state of rest. I add three series of observations

on the moments in the keeper, whose nearest end is 78.42^{mm} from the magnet, by which it will be seen how very unequal the distribution may be on each side of the point of indifference, even when the greatest moment was constant. The last of the three series was made at a slower rate than the two first, and the keeper was gently tapped with a piece of wood, each time the magnet was placed opposite it, and before its removal.

Dist. fr. nearest end of keeper to magnet in millim.^{rs}	5.5^{mm}	15	25	35	50	65	80	95	110	125	140.5
1st Series.	45	53	57	58	57	55	52	47	41	31	19
2nd Series.	49	54	57	58	58	57	55	51	49	42	16
3rd Series.	48	54	57	58	53	41	31	24	18	13	7

I found that if the magnet is brought to its position in front of the keeper from some distance, the moment in the latter is always smaller in the part remote from the magnet than when the magnet is brought from a shorter distance.

From the tabular comparisons of the magnetic distribution (page 52—61) in connection with the curves of the moments, it appears that as the distance between the magnet and keeper decreases, the moments increase regularily in the keeper; whereas in the magnet such increase is but very slight, so long as the distance is greater than about one diameter; but when less, the moments become perceptibly influenced by any change in the distance, and increase near the keeper rapidly as that distance decreases, so that when it becomes zero, the moments have risen in the plane of contact to several times their original magnitude.

The free magnetism of the keeper, as well on the side as on the end-planes, increases gradually as the distance from the magnet decreases; in the magnet on the other hand, the *total* free magnetism increases so slightly, that at contact, it has become but a small fraction greater than at a distance altogether beyond the influence of the keeper. But at the same time, a very considerable change in the *distribution* of the free magnetism takes place in the magnet, which as the keeper approaches, is always diminished on that half of the side, nearest to the keeper, while

it increases slightly on the other half. It also increases simultaneously on the end-plane nearest the keeper, but remains entirely or almost entirely unchanged on the further end-plane.

While the magnetic moment in the keeper is largest nearest the magnet when in contact, and decreases almost evenly towards the further end, this relation is gradually altered at distances greater than about $^2/_3$ radius; for the moments then increase from the end towards a point nearer the middle, but diminish again from this point towards the further end, where it is always smallest. This causes the free magnetism, (which at contact or at short distances between magnet and keeper, is almost evenly distributed over the whole side of the keeper) at increasing distances to decrease very rapidly on the part nearest to the magnet and entirely to disappear, when the distance between magnet and keeper is about $^2/_3$ of the radius. At a point therefore between the front and the centre, the free magnetism is then greatest, and decreases again from this point towards the back.

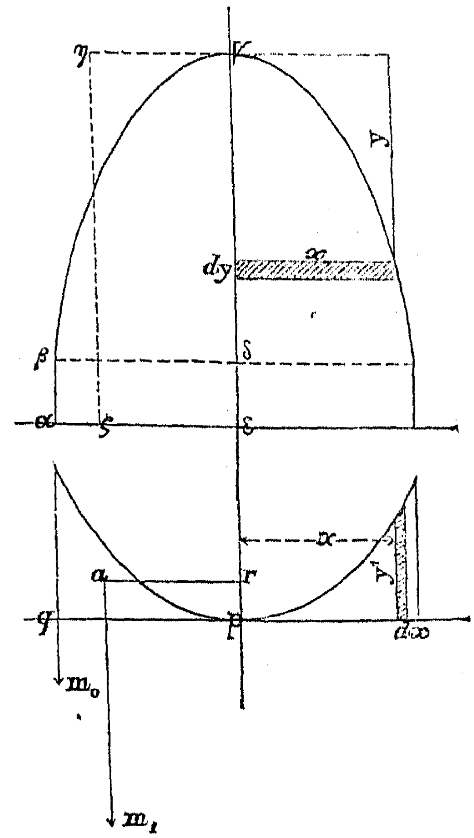
The distribution continues to vary at still greater distances. Instead of there being at the distances hitherto considered — within about $^2/_3$ radius — only one kind of free magnetism spread over the side of the keeper, both kinds now appear with a point of indifference between them. This point is first formed at the end nearest the magnet, but as the distance increases, it gradually recedes towards the middle of the bar; the nearest portions of the magnet and keeper have then heterogeneous magnetism, and consequently attract one another. On the whole the distribution of free magnetism in a keeper approaches more and more, as the distance increases, to the state existing in a magnet when the opposite pole of another magnet is turned towards it.

The neutral point in a magnet, on both sides of which the heterogenous magnetism are distributed, is not constant; on the approach of a keeper, it moves towards that keeper.

It follows from the previous representation, that the distance between the two *poles* — the magnetic centres of gravity —of a magnet is equal to the base line of a rectangle, whose area is equal

to that of the curve of the moments, and whose height is equal to the largest ordinate of moments.

The point of application of the free side-magnetism $m_{,}$ is situated at the centre of gravity a of the figure representing the free magnetism and therefore at the distance ar from the middle; m_0 is the free end-magnetism at the distance $q\,p$; consequently, half the polar distance



$$x = \frac{m_{,}\; a\, r + m_0\; q\, p}{m_{,} + m_0}$$

Now $m_{,}\, a\, r + m_0\, q\, p$ denotes the area of half the moment-curve, and $m + m_0$ is equal to the largest ordinate of that curve; for as the statical moment of a curve of differences is represented by the area of the primary curve from which it is derived[1]), $m_{,}\, ar$ = area $\beta\ \gamma\ \delta$; and $m_0\, q\, p = \alpha\ \beta\ \delta\ \varepsilon$. The sum of the two moments is consequently $= \alpha\ \gamma\ \varepsilon =$ rectangle $\zeta\eta\gamma\varepsilon$ of same area.

According to the knowledge we possess with regard to free magnetism, a magnet may be considered as consisting of an infinite number of infinitely thin magnetic discs or strata, in which, reckoning from the North to the South end, the negative magnetism of the preceding disc does not completely saturate the positive magnetism of the disc behind it, except at the equator of the magnet, where consequently the point of indifference is situated. The unsaturated remnant, which is the free magnetism spread over the disc, influences the keeper in the same

[1]) According to the definition $y' = \dfrac{dy}{dx}$

$$\text{the moment } x \,.\, y'\, dx = x.\ \frac{dy}{dx} \,.\, dx = x\ dy$$

which last expression denotes an element of the area of the primary curve. The integral of the moments x dy on both sides of $\gamma\ \delta$ equals the area of the whole of that curve.

manner as the free magnetism of the terminal surfaces, and consequently the total attractive force, is equal to the sum of the individual influences exercised by all the elementary discs of the magnet and keeper. The mutual attraction of the molecular magnetic poles is inversely proportional to the square of their distances, but discs consisting of such molecules attract one another, as already mentioned, in a very different proportion. These attractions are proportional to the sum of the attractions between all the molecules, multiplied by the *cosines* of their angles of direction and divided by the squares of their respective distances. As above mentioned, I have calculated the attraction of two discs, parallel and opposite to each other, and of uniform density throughout, at distances between $^1/_{1000}$ and 20 radii. By multiplying the product of the masses of the discs — in the present case their free magnetism — by the calculated coefficients or moduli, the attraction at a given distance is determined between two discs of equal or unequal magnetic intensity, but whose magnetism is evenly distributed. The attraction between magnet and keeper, at those twelve distances for which the magnetic distribution was examined, has been calculated on this principle, the magnet having been considered as consisting of discs 1^{mm} thick, at and near the positive end, and of greater thickness up to 12^{mm}, according as they are situated nearer the negative end. The free magnetism for each stratum was determined by means of the curve of moments.

The keeper was also dealt with as consisting of strata, increasing in thickness from 1^{mm} to 12^{mm}, according as they were more remote from the magnet, and the free magnetism of each was found in the same manner as that in the strata of the magnet, and also considered as collected in a plane between the front and back of the stratum.

The combined effect was determined separately for the four principal parts of the magnet and keeper, i. e.

α, for the positive lateral portion; β, for the negative lateral portion γ, for the positive terminal surface; δ, for the negative terminal surface, but as mentioned before the negative lateral magnetism (β) is wanting in the case of the keeper, when at a short distance from the magnet.

The results of these calculations are given in the following table:

The effect of the different parts of

Distance between Magnet and Keeper in millimeters	0.284	0.795	1.690	3.373
Attraction between Magnet's positive terminal surface and				
The nearer (negat.) term. surf. of Keeper	+ 296613.0	+ 157261.0	+ 71194.0	+ 24796.0
The negative lateral portion - -	—	—	—	—
The positive lateral portion - -	— 8287.0	— 3563.0	— 1213.0	— 214.7
The posit. terminal surface - -	— 18.7	— 13.5	— 9.3	— 6.1
Σ	+ 288307.3	+ 153684.5	+ 69971.7	+ 24525.2
Attraction between Magnet's positive lateral portion and				
The negative terminal surf. of Keeper	+ 16259.0	+ 15970.0	+ 14194.0	+ 8919.0
The negative lateral portion - -	—	—	—	—
The positive lateral portion - -	— 1809.0	— 820.0	— 460.0	— 275.3
The posit. terminal surface - -	— 11.9	— 13.3	— 13.4	— 12.6
Σ	+ 14438.1	+ 15136.6	+ 13720.6	+ 8631.1
Attraction between Magnet's negative lateral portion and				
The negative terminal surf. of Keeper	— 449.3	— 336.3	— 235.31	— 152.91
The negative lateral portion - -	—	—	—	—
The posit. lateral portion - -	+ 229.2	+ 119.7	+ 76.70	+ 48.31
The posit. terminal surface - -	+ 6.6	+ 5.7	+ 4.65	+ 3.80
Σ	— 213.5	— 210.9	— 153.96	— 100.80
Attraction between Magnet's negative terminal surf. and				
The negative terminal surf. of Keeper	— 72.33	— 56.79	— 41.42	— 29.89
The negative lateral portion - -	—	—	—	—
The posit. lateral portion - -	+ 43.68	+ 24.32	+ 16.22	+ 11.21
The posit. terminal surface - -	+ 1.59	+ 1.42	+ 1.25	+ 1.04
Σ	— 27.06	— 31.05	— 23.95	— 17.64
$\Sigma\Sigma$	+ 302505	+ 168579	+ 83514	+ 33038

The above sums ($\Sigma\Sigma$) of the mutual attraction of the several parts, should now express the relation between total attractions, at the respective distances of the magnet and keeper, if all the attracting elements have been taken into account and the sup-

Dist.	0.284	0.795	1.690	3.373	5.110	7.620
$\Sigma\Sigma$	302505	168579	83514	33038	17766	8479
A	6486	3168	1459	598	331	166
$\frac{\Sigma\Sigma}{A}$	46.6	53.2	57.2	55.2	53.7	51.1

the magnet and keeper on each other.

5.110	7.620	10.96	18.24	29.39	39.21	56.54	78.42
+ 12709.0	+ 5116.60	+ 2232.00	+ 601.70	+ 157.40	+ 64.83	+ 23.76	+ 6.694
+ 210.5	+ 441.61	+ 203.01	+ 72.54	+ 25.75	+ 18.93	+ 10.12	+ 3 866
— 106.7	— 51.97	— 39.84	— 17.87	— 9.92	— 5.56	+ 3.42	— 1.320
— 5.0	— 3.93	— 3.28	— 2.50	— 1.82	— 1.43	— 0 96	— 0.652
+ 12807.8	+ 5502.3	+ 2391.9	+ 653.87	+ 171.41	+ 76.77	+ 29.50	+ 8.588
+ 5100.0	+ 2959.01	+ 1843.02	+ 657.70	+ 224 80	+ 107.20	+ 46.02	+ 14.710
+ 174.8	+ 229.10	+ 162.00	+ 97.79	+ 52.26	+ 28.89	+ 17.70	+ 12.630
— 197.4	— 126.41	— 72.03	— 41.64	— 24.85	— 15.78	— 12.50	— 6.518
— 12.2	— 10.60	— 9.52	— 8.16	— 6.76	— 5.48	— 4.26	— 3.290
+ 5065.2	+ 3052.1	+ 1923.5	+ 705.69	+ 245.45	+ 114.83	+ 46.96	+ 17.532
— 126.61	— 85.48	— 62.55	— 39.08	— 20.65	— 12.91	— 7.771	— 3.350
— 6.11	— 8.91	— 8.30	— 7.24	— 5.97	— 4.50	— 3.540	— 2.832
+ 38.94	+ 26.50	+ 18.47	+ 11.02	+ 6.88	+ 4.45	+ 3.298	+ 1.579
+ 3.43	+ 2.81	+ 2.25	+ 1.71	+ 1.25	+ 0.89	+ 0.670	+ 0.480
— 90.33	— 65.08	— 50.13	— 33.54	— 18.49	— 12.07	— 7.343	— 4.123
— 24.75	— 16.01	— 12.570	— 7.870	— 4.556	— 3.021	— 1.820	— 0.824
— 1.23	— 1.79	— 1.801	— 1.782	— 1.434	— 1.091	— 0.883	— 0.599
+ 8.61	+ 5.99	+ 4.358	+ 2.619	+ 1.724	+ 1.143	+ 0.876	+ 0.436
+ 0.96	+ 0.82	+ 0.730	+ 0.587	+ 0.443	+ 0.351	+ 0.330	+ 0.195
— 16.41	— 10.99	— 9.283	— 6.446	— 3.823	— 2.618	— 1.497	— 0.792
+ 17766	+ 8479.0	+ 4255.9	+ 1319.5	+ 394.55	+ 176.91	+ 67.620	+ 21.205

positions be correct, on which the calculation is based. If the sums therefore be divided by the measured attractions, corresponding to the distances, all the quotients should be equal. The case, however, is different. In the series below, — A indicates the measured attractions.

10.96	18.24	29.39	3921	55.54	78.42
4256	1320	394.6	176.9	67.62	21.205
85.2	27.3	8.49	3.77	1.48	0.455
50.1	48.3	46.8	46.9	45.7	46.6

These quotients so far from being equal, increase at first with the corresponding distances till they reach a maximum, at the distance of about $^5/_8$ radius; and then decrease, until at the distance of from 3 to 4 radii they are about $^1/_6$ smaller than at the maximum; after which at greater distances they remain almost unaltered; or in other words, commencing at the greater distances, the measured power of attraction, compared with that calculated, decreases as the magnet and keeper are brought nearer to each other, until at the distance of about $^5/_8$ radius it reaches a minimum, and thereupon again increases with the shorter distances.

Now, as the calculations are based on facts, respecting which no doubt exists, either as regards the attraction between magnetic molecules, — and from this the deduced attraction between the magnetic discs — or as regards the relation between the magnetic molecular moments and the free magnetism, — the difference existing must be either owing to the fact, that all the terms have not been included in the calculations, or that in some respect or other, the suppositions started from have been erroneous.

With respect to the first possibility, it seems hardly conceivable that there should be any attractive forces to consider, beyond those already taken into account; and as to the second, there is probably only one term whose correctness would admit of any doubt, or rather whose treatment in the calculation might be conceived defective, namely that connected with the *density* of the free magnetism spread over the elementary discs, which in the calculation has been treated as if uniform. It was therefore necessary to ascertain, whether the cause of disagreement between the calculated and actual attraction should be sought for in this circumstance.

Any one who has occupied himself with these matters, will probably have observed the increased attraction which appears on the edges and corners of a magnet. It is also well known that on examining the several magnets in a magnetic magazine or battery, which has been joined for any length of time, those near the centre will be found to have become considerably weakened, and

at times, they may even be found to have lost most of their magnetism. This circumstance arises from the action of the magnetic molecules upon one another, causing a tendency of the axes to take up a position reverse to that of the adjacent molecules. The surface-molecules of a magnet are only acted upon from one side, by the adjacent molecules of the same transverse section; the inner ones from both sides, and with a force increasing towards the centre. The demagnetizing action is greatest here on this account, and causes not only a temporary but a permanent weakening of the magnetic moment. The interior layers of a magnetic magazine or battery can therefore, after a length of time and under certain circumstances, be not only completely demagnetized, but their polarity may even become reversed (*Wiedemann* p. 356). It seems therefore probable that what is the case in this instance may also happen in a single magnet, namely that the exterior layers are more intensely magnetic than the interior, and especially in the terminal surfaces where the free magnetism is greatest. And it was thought possible that this might be the case to such an extent, as to influence materially the phenomena of attraction.

The following investigations were performed in order to arrive at some positive knowledge on the subject. A preliminary trial proved that when the rounded end of a soft iron cylinder 4^{mm} in diameter was brought in contact with the centre of the end-surface of a magnet, 10.5^{mm} in diameter, it was attracted with only $^7/_8$ths of the force by which it was attracted when at the edge or circumference.

But this relation does not really show the difference between the free magnetism on the two above mentioned parts of the surface; for even if there had been no free magnetism at the centre, before the cylinder was brought in contact, the cylinder would cause the magnetism from other parts of the bar to accumulate at this point. It is therefore probable that the difference in the free magnetism was greater than the above experiment showed. If the terminal surface of the magnet be completely covered by an iron plate, no such local induction will take place.

A quantity of free magnetism will be induced on the plate, and the distribution on the several points of the surface will very nearly correspond to that existing on the points opposite in the magnet.

A hole, 3^{mm} in diameter, was bored at one end of a bow of soft iron, such as that represented at page 33; the plane ends of which compassed and covered the terminal surfaces of the magnet, and thus acted as a kind of keeper.

The hole could be placed opposite any point, between the edge and the centre of the magnet's end-surface, and an iron cylinder, fitting loosely into it, could thus be brought in contact with any of the points. The cylinder was held fast by the magnetism, if placed on or near the edge; but when it was drawn near to the centre, only a faint attraction was perceptible.

By inserting the magnet in iron filings, it was clearly shown, that the magnetism was strongly gathered on and very near the edge, where the filings were closely grouped together like a ring-shaped brush; whereas they were but thinly distributed over the middle.

The case was the same with the keeper. It was placed in a vertical position, iron filings were then strewn over the surface at the upper end, and a magnet placed at some distance over it. By tapping the keeper lightly with a piece of wood, the filings worked their way to the circumference, where they assumed a cup-like form, and the inner surface was left nearly bare. This was the case, when the magnet was even, as much as several inches off. If on the contrary the keeper was very near the magnet, on tapping it as before, the filings ranged themselves in points, like ends of thread, up towards the magnet; and as, on account of their proximity to the magnet, they naturally became far more magnetized than the surface of the keeper, they induced increased magnetism at the points of contact, which held them fast and made it more difficult for them to move towards the circumference. The same grouping of the filings took place on the keeper at the end farthest from the magnet, even when the keeper was very

short and pointed towards the centre, at the end touching or near to the magnet, so that the induction emanated from the centre.

Although the above experiments must be decisive as to the unequal distribution of magnetism over the surface, it was still considered of some interest to make a further investigation into the matter.

A magnet was made for the purpose, of the same dimensions as those used for the other experiments, viz. 146^{mm} long and 10.5^{mm} in diameter. After it had been magnetized to saturation, it was left for several days without any armature, and exposed to frequent shocks, in order to deprive it of the magnetism it had a tendency to lose. Its power of attraction was then carefully measured, and after the bar had been left for two days encompassed by an iron bow, as before shown, its attractive force was again measured. No loss of magnetism was then apparent. Its molecular moments were then measured and gave the following result:

Distance between middle of coil to North pole-end	Deviation	Coil's distance from South pole-end	Deviation
5^{mm}	4.46	5^{mm}	4.55
18	8.25	18	8.42
31	11.09	31	11.30
44	13.07	44	13.22
57	14.25	57	14.60
64	14.77	64	14.89
71	14.85	71	15.04

The magnet was then again placed in the bow, and after being well coated with varnish about the joints, it was immersed in dilute muriatic acid, which was maintained at a temperature of 40° Centigrade. Before immersion the weight was taken and found to be 105 Gr., but after a lapse of 15 days it was found to be reduced to only 76 Gr. The diameter was therefore 8.925^{mm} and the loss 0.274 of the original volume. The magnetic moments were again examined and found :

Dist. fr. N. P.	Deviation	Dist. fr. S. P.	Deviation
5	2.60	5	2.68
18	5.00	18	5.18
31	6.70	31	6.98
44	7.50	44	8.15
57	8.50	57	8.93
71	9.05	71	9.20

The magnet was again put into the bow, immersed in the acid and treated as before. After a lapse of 11 days, its weight was reduced to 56 Gr. The diameter was therefore 7.66^{mm}, and the loss of volume 0.467. The molecular moments were:

Dist. fr. N. P.	Dev.	Dist. fr. S. P.	Dev.
5	2.02	5	2.16
18	3.65	18	3.86
31	4.95	31	5.33
44	5.85	44	5.90
57	6.40	57	6.45
71	6.70	71	6.70

After again being subjected to the same treatment in muriatic acid during 8 days, the weight was 42 Gr.; its diameter consequently 6.61^{mm}, and the loss of volume 0.60. The molecular moments were now as follows:

Dist. fr. N. P.	Dev.	Dist. fr. S. P.	Dev.
5	1.57	5	1.48
18	2.85	18	2.81
31	3.65	31	3.72
44	4.29	44	4.26
57	4.74	57	4.59
71	4.88	71	4.81

The magnet was for the fourth time immersed in the acid, and taken out after a lapse of 8 days, when its weight had decreased to 27 Gr., or to about one fourth of its original weight. The diameter was therefore 5.35^{mm}, and the loss in volume 0.743. The deviations for the molecular moments were:

Dist. fr. N. P.	Dev.	Dist. fr. S. P.	Dev.
5	0.98	5	1.07
18	1.74	18	1.89
31	2.34	31	2.45
44	2.70	44	2.90
57	3.09	57	3.05
71	2.99	71	3.07

Finally the magnet was once more immersed in the acid, and removed after 8 days. It now weighed 19.5 Gr., or something less than one fifth of its original weight. The loss of volume was then in all 0.814, and the diameter 4.51^{mm}. The deviations for the molecular moments were:

Dist. fr. N. P.	Dev.	Dist. fr. S. P.	Dev.
5	0.60	5	0.72
18	1.19	18	1.26
31	1.60	31	1.69
44	1.88	44	1.94
47	2.01	57	2.04
71	2.04	71	2.06

From these observations the following relation may be deduced, when the magnet's original diameter, the volume corresponding to it and the original total free-magnetism in each half of the bar is represented by unity:

Diameter	1.00	0.89	7.77	0.66	0.53	0.45
Volume.	1.000	0.726	0.533	0.400	0.257	0.186
Total free magnetism.	1.000	0.611	0.447	0.325	0.204	0.137
Aver. magn. on termin. surfaces .	0.186	0.115	0.087	0.067	0.046	0.033
Do. on the sides . . .	0.814	0.496	0.360	0.258	0.158	0.104

According to this the magnetism in the strata or crusts lying successively within each other is distributed as in the following table, in the first four columns of which all the elements of the original magnet are represented by unity, viz. the radius, volume, and the free magnetism of the end-surfaces and lateral parts, respectively 18.6 and 81.4 per cent of the total magnetism. In

the fifth column, the total free magnetism per volume in the 1st — exterior — crust is represented by unity:

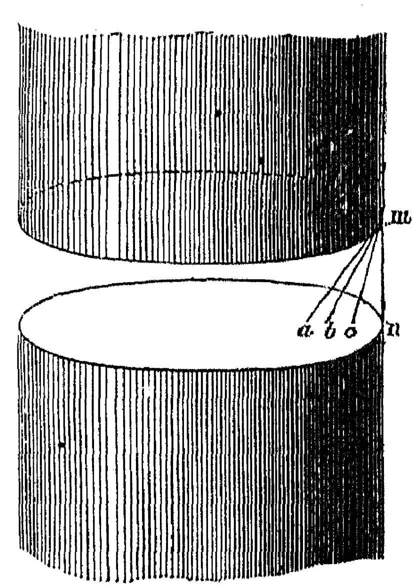
Stratum from surface	Thickness	Volume	Free magnetism		Total magn. per volume
			Termin. surf.	Lateral surf.	
1st	0.15	0.276	0.382	0.391	1.00
2nd	0.12	0.191	0.151	0.167	0.61
3rd	0.10	0.133	0.107	0.125	0.65
4th	0.12	0.143	0.113	0.123	0.60
5th	0.08	0.071	0.070	0.066	0.67
Inner kernel	0.43	0.186	0.177	0.128	0.52

Thus then we see again, that a greater quantity of magnetism is gathered round the circumference and in the lateral envelope, than nearer the axis of the magnet. But the difference has in reality been much greater than the above figures indicate, for as it is the exterior layers which weaken the moments of the interior ones, those of the latter will naturally be increased when the outer layers are removed, in so far as they have not been permanently weakened. As the diameter of the magnet has been diminished, the moments of the molecules, and the free magnetism depending on them, must therefore have been gradually increased; the measurements of the interior portions have consequently given greater deviations, than those corresponding to their moments, previous to the removal of the exterior crusts.

It follows from the series of observations, as well as from the graphic representation given Pl. II, Fig. 5, that the proportion according to which the free magnetism is distributed over the side and the end-surfaces, changes gradually, as the proportion between the length and the diameter changes, — that while for the magnet with diameter 10.5^{mm} it was distributed in the proportion of 81.6 : 18.6 or about as 1 : 0.22, the difference varies less as the diameter of the magnet becomes less, until for the remaining kernel, with 4.51^{mm} diam., it is as 10.4 : 3.3, or about as 1 : 0.32. The consequence of this is, that the end-surfaces of the remaining kernel have, in proportion to their area, very nearly as much free magnetism as those of the original magnet.

In conceiving the distribution of magnetism over a terminal surface, or indeed, any transverse section, we may regard the disk as composed of an infinite number of concentric rings, increasing in magnetic intensity from the centre, each having its own degree of magnetism evenly distributed; and, under certain circumstances, in order to calculate the *effect*, we may in a degree, without involving any considerable error, conceive the whole magnetism resolved into two parts — one, evenly distributed over the entire surface, the other, evenly collected in the periphery. On such an assumption the force of attraction between two disks, would have to be considered as the sum of three attractions, viz. 1, the attraction between the magnetism of the areas; 2, the attraction between the magnetism of each periphery and that of the area opposite, and 3, the attraction between the magnetism of the peripheries.

It is self evident, that for very short distances between the magnet and keeper, a calculation based on this assumption will not be applicable; it can only lead to a correct result, and that only approximately so, when the distances are so great that lines drawn from a point *m* in the periphery of the one, to points *a*, *b*, *c*, *n* in the corresponding radius of the other, are nearly of equal length, or, when consequently *ma*, *mb*, *mc* form with *mn* only small angles; when in short $\frac{A_a}{(\text{distance}\, am)^2} \sin man$ is approximately $= \frac{A_n}{(\text{distance}\, mn)^2}$, where A_a, A_n represent respectively, the magnetic force of the points *a*, *n* etc. with regard to *m*.



If the magnet and keeper be so near each other, that *m* forms with *n* and *a*, *b*, *c* . . . great angles, then the attractions in *a*, *b*, *c* exert an effect, perceptibly less in the direction of the axis, than when they are concentrated in *n* in the circumference; and so, applied to the shorter distances between magnet and keeper, a calculation based on the above

assumption would give results too large, increasing inversely with those distances, at a rapid rate.

The attraction between two circumferences varies with the distance, in a different proportion to the attraction between two areas, and to that between one circumference — a line — and one area.

As it has not been possible for me to find any calculation made of the variation of attraction, with regard to the distances, between two peripheries, and between one periphery and one circular area, I have been obliged to carry out such calculations for these figures also — and hope on some future occasion to have the honour of laying the results before this Society. It will here be sufficient to mention, that the attraction between a circular periphery and a circular area with the same radii, varies with the distances according to a law, similar to that for the attraction between two circular areas, — with this difference, however, that the increase of attraction in the former, as the distance decreases, is less than for the latter, so that the attraction at contact is only about half as great. In both cases the force at contact has a definite magnitude, and as the attracting objects are removed from each other, tends, assymtotically, to become inversely proportional to the squares of the distances.

The case is very different with regard to the attraction between two circular peripheries. Between the distances of zero and one radius (r) the attraction varies nearly inversely with the distance itself (in 1st power,) though somewhat less rapidly, in consequence of which, the product of the attraction and distance is slowly increasing from contact, till the distance is equal to r, when it reaches its maximum, being then about 6 per cent. greater than for the shortest distances. As the distances increase beyond r, the attraction decreases at a quicker rate, and tends gradually to become inversely proportional to the squares of the distances.

From what has now been stated, it is clear, that in order to obtain results which may be expected to agree with those obtained by actual measurement, a correction is required, or a modification

must be made in the former mode of calculating the attraction at varying distances. Instead of multiplying the free magnetism for all transverse sections of magnet and keeper with the module of attraction for two circular surfaces, as has been done, it must be multiplied by the sum of the moduli of the three attractions before referred to, each modulus being multiplied by a factor corresponding to the value of that one of the attractions to which it belongs, compared with the value of the other two.

The terminal surfaces possess a quantity of free magnetism, vastly preponderating over that of any of the other transverse sections. All the three modules of attraction tend, as the distances of action increase, to become inversely proportional to the squares of these distances, and consequently to vary at a *uniform* rate. And as the amount of the free magnetism of the sides appertains to transverse sections in the magnet, for the most part, at such distances from those in the keeper, that all the three modules vary approximately at the same rate, and as likewise, by the correction above mentioned, nothing more than an approximately accurate result can be attained, I have, in calculating this correction — to be applied to the results previously found — only carried it out for the terminal surfaces, for which alone it can have a value of any consequence.

In the following series the three moduli of attraction are given for those distances, for which the calculation of the attraction between the magnet and keeper has been made. A indicates the module for two circular areas, B for a circular periphery and a circular area opposite, and C for two circular peripheries.

Distance between term. surfaces		A	B	C
0.284^{mm}	= 0.0549 Radii	0.8645	0.4465	2.9139
0.795	0.1515	0.7165	0.4010	1.0587
1.690	0.3218	0.5451	0.3348	0.5000
3.373	0.6427	0.3528	0.2519	0.2599
5.110	0.9431	0.2403	0.1825	0.1730
7.620	1.451	0.1492	0.1224	0.1131
10.96	2.088	0.08817	0.07737	0.07209
18.24	3.493	0.03676	0.03451	0.03345
29.39	5.598	0.01528	0.01491	0.01465
39.21	7.467	0.00870	0.00856	0.00849
55.54	10.579	0.00440	0.00436	0.00434
78.42	14.934	0.00223	0.00221	0.00221

The question now to be considered is, how much of the free magnetism of the end-surface may be taken as being equally distributed over the surfaces, and how much concentrated in the peripheries alone. If both quantities are supposed as equal, and consequently, the one half as being evenly distributed over the surfaces, and the other collected in the circumferences, then the new modulus, wherewith the product of the total free magnetism of the surfaces of magnet and keeper has to be multiplied, becomes:

$$0.5^2\ A + 2.\ 0\ 5.\ 0.5\ B + 0.5^2\ C.$$

The first term of attraction, pages 74 and 75, or that which has regard to the mutual action of the terminal surfaces, must therefore be divided by that modulus of attraction (A) with which it has been multiplied, and instead, multiplied by the above mentioned new modulus. This first term for the

	mm		
Distance between magnet and keeper . . .	0.284	0.795	1.690
was	296613	157261	71194
After the correction it becomes	406292	141423	56012
whereby $\Sigma\Sigma$ becomes	412184	152741	68332
The measured attraction, A	6486^{mgr}	3168	1459
and $\frac{\Sigma\Sigma}{A}$	63.55	48.21	46.84
Differences over or under the mean, 46.55, in last 10 observations			+ 0.29

The quotients $\left(\frac{\Sigma\,\Sigma}{A}\right)$ thus obtained, by dividing the calculated forces of attraction by those measured for distances, within which the employed method of correction, according to what has been stated, may be considered applicable — are on the whole, as nearly constant as could have been expected, considering the many circumstances that may have caused discrepancies in the determination of the dividends, among which may be mentioned particularly the limited number of points for which the magnetic moments were measured, and upon which depend the curves for the distribution of the free magnetism.

In order to obtain more accurate curves, it would have been necessary to employ a greater number of points of observation on the part of the keeper near the magnet, and to make at least two observations for each of these points, but that would have required more time than I had at my disposal. It will be borne in mind also, that the observations extended over several months, during which time the magnet did not maintain its strength unchanged; — it must, however, be mentioned, that corrections for such changes were made whenever they could be substantiated.

It will be seen, that between the distance of about $^1/_3\ r$, and the greatest distances for which the carrying out of anything like reliable measurements have been possible, the disagreement between the quotients is small, the deviation from the mean being in all cases less than 2 per cent. of this mean, although the corresponding distances between magnet and keeper lie between the limits 1.69^{mm} and 78.42^{mm}, corresponding to 0.3218 and 14.93 radii, or

3.373	5.110	7.620	10.96	18.24	29.39	39.21	55.54	78.42
24796	12709	5116.6	2232.0	601.70	157.40	64.83	23.76	6.694
19580	10292	4348.0	1993.0	569.80	153.80	63.80	23.54	6.662
27872	15349	7710.0	4017.0	1288.8	391.0	175.90	67.60	21.173
598	331	166	85.2	27.3	8.49	3.77	1.48	0.455
46.60	46.37	46.45	47.15	47.17	46.06	46.67	45.67	46.54
+ 0.05	— 0.18	— 0.10	+ 0.60	+ 0.62	— 0.49	+ 0.12	— 0.88	— 0.01

are to each other as 1: about 46; and the attractions which appertain to these distances, lie between 1459 and 0.455 milligrams, and are thus to each other in the ratio of more than 3000 : 1.

In magnets whose polar surfaces possess an equal quantity of free magnetism, the molecular deflections are inversely proportional to the transverse areas of the magnets. It has previously been shown, that when a keeper that is not very long, is placed between the heterogeneous poles of two equally strong magnets, of the same diameter as the keeper, then all the transverse strata from end to end of the latter, will have almost equally great molecular moments, and there will be nearly no free magnetism on the side. The molecules of the consecutive strata, therefore saturate each other. If the equally strong magnets have different diameters, and the intervening keeper has the shape of a truncated cone, whose ends coincide with those of the magnets, a mutual saturation of the consecutive strata will take place; but that being the case, the deflection of the individual molecules, must necessarily increase from the large to the small end, in the same proportion as the transverse areas decrease.

If with magnets of different diameters the keeper be a cylinder, with the same radius as the larger magnet, something of the same kind will take place as when it is conical, since the direction of saturation between the molecular series, will be from the larger to the smaller magnet. The case is similar, when the keeper has the same diameter as the smaller magnet; the direction in which the front molecules of the larger magnet saturate those behind, diverge from the end-surface of the keeper, by which the front stratum is saturated. The quantity of magnetism collected here is very nearly as great, as if the keeper, as in the former case, were of the same diameter as the magnet. Hence it follows, that the sum of the molecular moments of the keeper is equal, or nearly so, in both cases, at the surface of contact, or that the

molecular deflection of the contact-areas of the keeper is inversely proportional to those areas.

The remarks that have just been made, respecting the case of a keeper, when placed between two magnets, apply also to where a keeper is placed with one end against a magnet, and the other end is free; whether the diameter of the keeper be the same, or greater, or within certain limits smaller, than that of the magnet, the quantity of magnetism saturated by it, at the surface of contact is nearly equally great.

To trace what follows as a consequence of this, let it be assumed, that the transverse area of the keeper equals half that of the magnet, then the terminal molecules have each a torsion approximately twice as great, as that which takes place, when the magnet and keeper have equal diameters. The attraction is proportional to the product of the *sines* of the angles of molecular deflection, and is consequently, about four times as great for each terminal molecule, in the present instance, as when the transverse areas of the magnet and keeper are equal. But as the number of molecules is only half as great, the entire attraction of the terminal surfaces should be twice as much, as when the large keeper touches the magnet. It is true that this proportion does not really exist to its full extent, and it will be seen from the following series of observations and from the tables of attraction, how the moments and forces are modified; but by the above reasoning an explanation is given of the remarkable fact, that the attraction of a keeper within certain limits smaller than the magnet, is greater, and sometimes considerably greater, than that of a larger keeper towards the same magnet.

Hence it also follows as a general rule, that when a certain magnetic power is concentrated on a small surface, the force of attraction is greater, than it is when distributed over a larger surface, and from this it will be understood also, why the attraction between a magnet and keeper, when the ends are slightly rounded, is greater, than when the terminal surfaces are plane; and why keepers for horseshoe-magnets are more strongly attracted

when the surfaces in contact with the pole-surfaces are slightly rounded, than when they are plane.

In the following series, the deviations for the molecular moments are given both for magnet and keeper, when the magnet 146^{mm} long by 10.5^{mm} diameter is brought in contact with keepers 146^{mm} long, with diameters respectively 13.0^{mm}, 10.5^{mm}, 8.0^{mm} and 5.5^{mm}, and when the magnet is brought opposite the three last keepers, at the distances of 0.284^{mm} and 5.100^{mm}:

Deviations for molecular moments

in a magnet, with keepers of various diameters.

Magnet and keeper, both 146^{mm} long 10.5^{mm} diameter, in contact.

Magnet		Keeper	
Distance	Deviation	Dist. fr. end oppos.magn	Deviation
5.5^{mm} fr.N.End	10.02	5.5	9.14
15	11.13	15	8.42
30	12.57	25	7.73
45	13.70	35	7.05
60	14.38	50	6.12
75^{mm} fr. S. End	14.56	65	5.12
60	14.11	80	4.22
45	12.58	95	3.33
30	10.20	110	2.51
15	6.86	125	1.72
5.5	4.35	140.5	0.90
The keeper removed 0.824^{mm}			
5.5^{mm} fr.NE.nd	8.74	5.5^{mm}	7.70
15	10.15	15	7.15
30	12.03	25	6.60
45	13.36	35	6.04
60	14.15	50	5.18
75^{mm} fr. S. End	14.37	65	4.36
60	14.01	80	3.58
45	12.53	95	2.78
30	10.07	110	2.10
15	6.78	125	1.42
5.5	4.33	140.5	0.81

The keeper removed 5.32^{mm}.

Magnet		Keeper	
Distance	Deviation	Distance	Deviation
5.5^{mm} fr. N. End	4.86	5.5	2.74
15	7.50	15	2.80
30	10.25	25	2.76
45	12.12	35	2.64
60	13.24	50	2.35
75^{mm} fr. S. End	13.68	65	2.03
60	13.55	80	1.69
45	12.20	95	1.28
30	9.87	110	1.00
15	6.61	125	0.72
5.5	4.20	140.5	0.49

Magnet 10.5^{mm} diameter keeper 5.5^{mm} diam.
In contact

5.5^{mm} fr. N. End	9.06	5.5	7.95
15	10.43	15	7.32
30	12.14	25	6.66
45	13.47	35	6.05
60	14.23	50	5.14
75^{mm} fr. S. End	14.46	65	4.29
60	14.08	80	3.42
45	12.58	95	2.60
30	10.12	110	1.90
15	6.80	125	1.26
5.5	4.34	140.5	0.70

The keeper removed 0.284^{mm}

5.5^{mm} fr. N. End	7.24	5.5	5.54
15	9.13	15	5.18
30	11.37	25	4 83
45	12.86	35	4.39
60	13.82	50	3.73
75^{mm} fr. S. End	14.16	65	3.12
60	13.81	80	2.56
45	12.36	95	1.96
30	10.06	110	1.49
15	6.71	125	1.00
5.5	4.30	140.5	0.53

The keeper removed 5.32mm.

Magnet		Keeper	
Distance	Deviation	Distance	Deviation
5.5mm fr. *N.* End	4.53	5.5mm	2.03
15	7.24	15	2.02
30	10.06	25	1.93
45	12.00	35	1.82
60	13.16	50	1.60
75mm fr. *S.* End	13.65	65	1.39
60	13.51	80	1.10
45	12.17	95	0.81
30	9.84	110	0.61
15	6.60	125	0.39
5.5	4.20	140.5	0.42

Magnet 10.5mm diam. keeper 8.0mm diam.

In contact

5.5mm fr. *N.* End	9.64	5.5mm	8.64
15	10.83	15	7.97
30	12.42	25	7.34
45	13.62	35	6.68
60	14.30	50	5.75
75mm fr. *S.* End	14.54	65	4.81
60	14.13	80	3.93
45	12.56	95	3.06
30	10.13	110	2.32
15	6.80	125	1.56
5.5	4.34	140.5	0.83

The keeper removed 0.284mm.

5.5mm fr. *N.* End	7.96	5.5mm	6.75
15	9.64	15	6.20
30	11.70	25	5.85
45	13.11	35	5.35
60	14.00	50	4.60
75mm fr. *S.* End	14.32	65	3.95
60	13.91	80	3.23
45	12.43	95	2.51
30	10.07	110	1.90
15	6.83	120	1.27
5.5	4.32	140.5	0.70

The keeper removed 5.32^{mm}.

Magnet		Keeper	
Distance	Deviation	Distance	Deviation
5.5^{mm} fr. *N.* End	4.70	5.5^{mm}	2.38
15	7.38	15	2.40
30	10.16	25	2.37
45	12.06	35	2.27
60	13.20	50	2.04
75^{mm} fr. *S.* End	13.68	65	1.75
60	13.53	80	1.44
45	12.18	95	1.13
30	9.85	110	0.85
15	6.60	125	0.58
5.5	4.20	140.5	0.39

Magnet 10.5^{mm} diam. keeper 13.0^{mm} diam.
In contact

5.5^{mm} fr. *N.* End	10.33	5.5^{mm}	9.39
15	11.34	15	8.67
30	12.70	25	7.94
45	13.81	35	7.27
60	14.46	50	6.30
75^{mm} fr. *S.* End	14.62	65	5.28
60	14.17	80	4.33
45	12.63	95	3.42
30	10.22	110	2.55
15	6.87	125	1.75
5.5	4.35	140.5	0.99

The above deviations for the keeper of 13.0^{mm} diameter, could not be obtained by actual observation, as it was too large for the induction-coil used. However, the curve corresponding to the moments could certainly be determined, without appreciable error; the starting point at zero being given by the observations for the magnet, and as from this, the other points could be very nearly determined from the curve for the 10.5^{mm} keeper at contact, Pl. V, the deviations cited are arrived at by this means. As the moment-curves for the keeper partly intersect each other, they, and those of the magnet, are for the sake of distinction indicated on Plate V

partly by continuous, and partly by dotted lines of various kinds. The continuous lines indicate the moments, when the magnet and keeper of equal diameter (10.5^{mm}) act upon each other; the dotted lines . . . . when the keeper is 13.0^{mm}; those marked - - - - - - - - - - - - - - when the keeper is 8.0^{mm}; and those —.—.—.—.— when the keeper is 5.5^{mm} diameter.

We have hitherto only dealt with the phenomena of attraction, which take place between a permanent steel magnet and a keeper of soft iron. When two permanent steel magnets are employed, the relations are different.

We have seen that the distribution of the moments in the magnet is materially different from that of the moments in the keeper. The distance between magnet and keeper has but little effect on the *quantity* of free magnetism in the former, while in the latter it is solely dependent upon it.

At short distances, most of the magnetism is collected on the terminal surface, of the magnet nearest the keeper, and only a small quantity on the side, but if the keeper is withdrawn, this distribution changes rapidly, so that at a distance equal to the diameter of the magnet, the relation is reversed, and it is now at the side, that the greatest quantity is found, while at the end there is merely a fractional portion. Beyond this distance, the keeper has but a slight influence, either on the distribution or the quantity of free magnetism. In the keeper on the contrary, the free magnetism is always dependent on the distance, which, if it is short, has but little influence on the relative distribution. The entire quantity of one kind of magnetism is collected on the terminal surface nearest the magnet, and the other kind distributed over the surface on all the other parts of the keeper. But when the distance is greater than $1/3$ of the magnet's diameter, both kinds of magnetism arise on the side of the keeper also, and the distribution becomes gradually more like that in the magnet, as the distance is increased. As the quantity of magnetism in a keeper always

decreases, when the distance from the magnet increases, so, the attraction diminishes rapidly with the distance.

The distribution of the moments and free magnetism in a magnet, is influenced in the same way by another magnet, as by a keeper, but as the reaction from a magnet is stronger, and more slowly diminished by distance, the alterations effected in the opposite magnet by changes of distance are less rapid, than those produced by a keeper, and the mutual action of two magnets, on each other, is therefore more constant. The results of observations of the moments in two magnets, while in contact, and at five different distances from each other, are given in the subjoining tables, and a graphic representation is shown on plate VI. Both the magnets employed, **A** and **B**, had on and about the two terminal surfaces facing each other almost exactly the same quantity of free magnetism, whereas the moments towards the middle, and therefore the total free magnetism, is somewhat less in **B** than in **A**, which is probably the result of the unequal tempering of the steel in **B**, as mentioned in connection with certain of the other experiments.

Deviations for the moments in the magnets A and B,

in contact and at unequal distances from each other, when the South Pole of A is turned towards the North Pole of B.

The magnets in contact.

Distance from surfaces of contact.	A	B
5.5mm fr. n. e.	12.60	12.51
15	13.18	12.76
25	14.17	13.44
40	15.11	14.37
55	15.64	14.77
70	15.31	14.27
70 fr. remote ends	14.96	13.88
55	13.77	12.52
40	11.95	10.73
25	9.48	8.44
15	7.14	6.18
5.5	4.08	3.70

When removed 0.288^{mm} from each other

Distance	A	B
5.5^{mm} fr. near. ends	11.14	11.05
15	12.21	11.94
25	13.38	12.86
40	14.62	13.98
55	15.31	14.59
70	15.12	14.12
70 fr. remote ends	14.79	13.76
55	13.65	12.41
40	11.86	10.66
25	9.42	8.39
15	7.11	6.15
5.5	4.06	3.68

When removed 1.72^{mm}

5.5^{mm} fr. near. ends	8.25	8.22
15	10.14	9.94
25	11.73	11.44
40	13.55	13.06
55	14.58	14.09
70	14.78	13.89
70 fr. remote ends	14.52	13.53
55	13.35	12.18
40	11.68	10.50
25	9.31	8.28
15	7.06	6.10
5.5	4.02	3.63

When removed 5.32^{mm}

5.2^{mm} fr. near. ends	6.30	6.29
15	8.63	8.48
25	10.75	10.36
40	12.87	12.36
55	14.11	13.63
70	14.35	13.60
70 fr. remote ends	14.18	13.26
55	13.16	11.96
40	11.55	10.40
25	9.22	8.20
15	6.98	6.06
5.5	3.99	3.60

When removed 26.14mm

Distance	A	B
5.5mm fr. near. ends	4.51	4.50
15	7.25	6.95
25	9.56	9.12
40	12.09	11.58
55	13.56	13.20
70	14.00	13.35
70 fr. remote ends	13.88	13.10
55	12.96	11.83
40	11.35	10.30
25	9.09	8.13
15	6.90	6.03
5.5	3.94	3.57
When the ends are free		
5.5mm fr. near. ends	3.88	3.87
15	6.71	6.43
25	9.03	8.57
40	11.62	11.13
55	13.13	12.85
70	13.68	13.10
70 fr. remote ends	13.52	12.86
55	12.66	11.67
40	11.15	10.18
25	8.96	8.04
15	6.82	5.98
5.5	3.90	3.54

From what has been shown in the preceding pages, it is evident, how highly complicated the relation is, which exists between the distances and attractions of two magnetic bodies. It may therefore be well before concluding to recapitulate as briefly as possible, the principle features of the phenomena, which take place, whenever a magnet changes its position opposite another magnetic body, under whose influence it has been brought, and point out their immediate consequences.

Least complicated are those phenomena which occur, when two equally strong cylindrical magnets of equal dimensions, placed

with their heterogeneous poles opposite each other, are approached to, or removed from each other.

If it be first considered what occurs, when two magnets, — say **A** and **B**, before named, are removed from each other, for instance from the distance 0.288^{mm} to 26.14^{mm}, or about five times the length of their radius, — it will then be seen, by referring to the tables pages 96 and 97 and to Plate VI, that the free magnetism, which on both the nearest end-surfaces may, for the shorter distance be taken at 10.7, while that distributed on the nearest side-surfaces is respectively 4.7 and 3.9, — undergoes a gradual change, in such manner, that when the distance becomes 26.14^{mm}, it is at the end-surface reduced to 3.1, but on the sides increased to respectively 10.9 and 10.3. While the magnets have been removed, the relation between the quantity of free magnetism on the end and side-surfaces has been totally reversed; the former has in fact, from being $2\frac{1}{2}$ times greater, been reduced to about 3 times less than the latter.

By referring to Fig. 2, Pl. VI, for the free side-magnetism, it will be observed, that in the nearest halves of the two magnets, the points of application of the collected forces with regard to the corresponding points in the opposite magnet, — which points are situated somewhere between the end-surfaces and the centre of gravity of the respective figures — are, when the distance between the two magnets is 0.288^{mm}, considerably removed from the end-surfaces; and without attempting by calculation to determine their exact position, they may probably, without great error, from the form of the figure, be computed to be situated at about 10^{mm} = 1.9 radii, from the end-surfaces.

The two side-magnetisms may consequently be considered as acting on each other at the distance $= (2 \,.\, 10^{mm}) + 0.288^{mm} = (3.865$ rad.) and the end-magnetisms, at the distance $0.288^{mm} = (0.0549$ rad.) Were the latter now concentrated in one single point of the surface, and the side-magnetism likewise in single points of the magnet's transverse sections 10^{mm} from the ends, and were these points situated in the axis passing through both magnets, then the

mutual effect of the end- and of the side-magnetism, would have to each other the proportion: —

$$\frac{10.7^2}{0.288^2} : \frac{4.7 \cdot 3.9}{20.288^2} = 1380 : 0.0445 = 31000 : 1$$

But owing to the magnetism being distributed over the areas of the surfaces, and not concentrated in single points, the proportion becomes entirely different; agreeably to the table given in the paper before cited, on the attraction of circular areas, — Christiania Vidensk. Selsk. Forhandl. for 1875, pages 265 and 267 — the proportion becomes: —

$$0.8648 \cdot 10.7^2 : 0.03066 \cdot 4.7 \cdot 3.9 = 99.01 : 0.5619 = 176.2 : 1$$

If the points of application of the *side*-magnetism of the one, with regard to the *end*-magnetism of the other magnet, be computed at 8^{mm} distant from the end-surfaces, and if the forces were concentrated in single points, then the total attraction would be: —

$$\frac{10.7^2}{0.288^2} + \frac{10.7 \cdot 4.7}{8.288^2} + \frac{10.7 \cdot 3.9}{8.288^2} + \frac{4.7 \cdot 3.9}{20.288^2}$$

$$= 1380.00 + 0.7319 + 0.6074 + 0.0445 = 1381.38$$

The relation between the terms to be added would then be: —

$$31000 : 16.4 : 13.6 : 1.0$$

But as they are distributed over surfaces, these values become:

$$0.8648 \cdot 10.7^2 + 0.1332 \cdot 10.7 \cdot 4.7 + 0.1332 \cdot 10.7 \cdot 3.9 + 0.03066 \cdot 4.7 \cdot 3.9$$

$$= 99.015 + 6.699 + 5.559 + 0.562 = 111.83$$

The relation between the terms to be added is here: —

$$176.2 : 11.9 : 9.9 : 1.0$$

A comparision between the two *sums* 111.83 and 1381.38 cannot be made, as the distances in the terms of the last mentioned are expressed in millimeters, whereas those distances, from which the coefficients in the terms of the first sum referred to are calculated, are expressed in radii of the magnets.

It would be reasonable to expect, that as the magnets are moved further from each other, the points of application of the side-magnetism should recede further from the end-surfaces, towards the centres of gravity of the free magnetism. But as the figures which represent the distribution of the side-mag-

netism, at the same time entirely alter their form, their centres of gravity are also shifting their position, in consequence of which the points of application are first moving nearer to the ends, — while the distance between the magnets increases from 0 to about 2^{mm}, — after which they recede, until at great distances, or strictly at ∞, they coincide with the centres of gravity, and are then, on an average in the two magnets, about 20^{mm} distant from the ends.

Though the figure of the free magnetism for the distance 26.14^{mm} is not much different from that for ∞, yet on account of the comparatively short distance of 26.14^{mm} (4.978 rad.) between the ends, and owing to the form of the figure, the points of application for the mutual effect of the side-magnetism may probably not be taken at more than about 15^{mm} from the surfaces, and that for the *side*-magnetism, with regard to the *end*-magnetism of the opposite magnet, at not more than about 12^{mm}.

The free magnetism of the sides of A and B is, as mentioned, respectively 10.9 and 10.3, and that of the end-surfaces 3.1. The mutual effect of the end-magnetism should therefore bear the relation to the mutual effect of the side-magnetism, — if the forces were concentrated in single points along the axis, — of: —

$$\frac{3.1^2}{26.14^2} : \frac{112.3}{56.14^2} \text{ or } 0.3950 : 1$$

Owing to the distribution of the forces on the transverse sections, this proportion is however modified to 0.3783 : 1, whereas, at the distance of 0.288^{mm} between the magnets, it was as 176.2 : 1.

The total attraction would have been, if the forces were concentrated in single points: —

$$\frac{3.1^2}{26.14^2} + \frac{3.1 \cdot 10.9}{38.14^2} + \frac{3.1 \cdot 10.3}{38.14^2} + \frac{10.9 \cdot 10.3}{56.14^2} = 0.09487$$

but for reasons mentioned, it is modified to

$$0.01906 \cdot 3.1^2 + 0.00917 \cdot 3.1 \cdot 10.9 + 0.00917 \cdot 3.1 \cdot 10.3$$
$$+ 0.004313 \cdot 10.9 \cdot 10.3$$
$$= 0.1832 + 0.3098 + 0.2928 + 0.4842 = 1.2700$$

In the above, that attraction only has been considered, which the nearer halves of the magnets exert on each other;

the more remote halves do also by their action modify the results found, but in a very slight degree for the distances here considered, amounting in fact for the larger (26.14^{mm}) to only — 0.0106; for the shorter distance (0.288^{mm}) it is therefore, in comparison with the powerful action, which is exerted by the nearest end-surfaces, almost vanishing.

The relation between the attractions at the two distances is consequently as 111.83 : 1.2594, or as 88.8 : 1. Had the fact of the forces being distributed over the transverse sections, instead of being concentrated in the central point of these same sections, not modified the attractions, then the relation would have been as 1381.38 : 0.09487 = 14561 : 1, where, however, no notice is taken of the slight effect of the remote poles.

What has now been pointed out, is only a rough sketch of that which takes place, when the magnets alter their mutual positions within those limits, where such alteration causes the greatest disturbance of the molecular conditions, and in which the form of the magnets has the greatest influence on the attraction. If now a comparison be made between the true relation, — determined by accurate measurements of the attractions at the two distances here considered, — and that found by the above computation, it will be seen, that while this relation, according to the tables of attraction pages 125 and 126, is for the distances 0.288^{mm} and 26.14^{mm} as 110.1 : 1.25 or as 88.1 : 1, the computed relation is, as stated above, as 88.8 : 1.

This almost complete coincidence is of course accidental, for in the computation, neither the position of the points of application of the side-magnetism had been determined by calculation, nor has any reference been made to the uneven distribution of the magnetism on the end-surfaces. We will however shortly consider, to what extent these omissions might modify the above result. First with regard to the effect, that an error in the determination of the distance between the points of application of the side-magnetism and the end of the bar might produce, it will be found page 74 that when the magnets are removed from each other only 0.288^{mm},

the terms, referring to the side-magnetism, are so small, that any error within reasonable limits, in the supposed situation of the points in question, will have only a very slight effect. But at the distance 26.14^{mm}, these terms are the most important, and any error here has comparatively great effect. Supposing then the distances assumed, respectively 15^{mm} and 12^{mm}, to be too great, and that they should be for instance respectively 12^{mm} and 9.5^{mm}, then the attraction would be 1.4594—0.0106 instead of 1.2700 - 0.0106, and the relation between the measured attractions at the distances in question to the computed ones as 88.1 : 77.1. Were the points of application on the contrary, at too short a distance from the ends, and should they more correctly have been, for instance, 18^{mm} and 15^{mm} respectively, then the attraction would have been 1.0991—0.0106 = 1.0885, and the relation between the measured and computed attractions as 88.1 : 101.7.

In both these cases the difference between the two is no large fraction, and yet, the points of application are supposed to have been moved a distance on either side of the position first determined, of which there can be no doubt, that it far exceeds the extent of possible error in the position first fixed upon.

With reference to the effect of the unequal distribution of the magnetism on the end-surfaces, which has not been taken into account, it will be observed, by referring to the series of quotients pages 74 and 75, that it so happens, that for the distances 0.284^{mm} and 29.39^{mm}, which are not very different from those, for which the above computations have been made; the relation between the measured and calculated results are about equal, even before any correction was made for the unequal distribution, being respectively 46.6 and 46.9. It is true, that the case referred to at pages 74 and 75 is not quite equal to the one of which we have spoken above, in as much as, at the shortest distance (0.284^{mm}), the nearest side-magnetism of one of the components of the attractive pair repels, whereas in the other case it attracts the side-magnetism of the other component; but at the above distance, the whole mutual effect of the two nearest side-magnetisms is comparatively

speaking very slight. At the greater of the two distances (26.14^{mm}) the side-magnetism of both components, attract each other in both the cases compared.

When the two magnets are removed still further from each other, very little change takes place in the moments, and the free magnetism remains therefore nearly constant; the circumstance that it is distributed on the end-surfaces, and not concentrated in a single point, exerts also less and less effect, as the distances increase. It has been already mentioned, that at distances not greater than the greatest of those hitherto considered, the more remote poles do only to a very slight degree modify the attraction of the nearer ones. It follows from this, that as the magnets are removed beyond 5 or 6 radii from each other, — without these distances however being greatly exceeded, — the attraction varies nearly as the sum of the products of the now almost constant forces, divided by the squares of their respective distances.

The end-magnetisms, whose points of application are nearer to each other, than those belonging to the other terms, exert consequently an attraction very nearly inversely proportional to the square of their distance from each other, but with regard to the other forces, they of course vary at a much slower rate than inversely as the squares of the distances between the *end-surfaces*, for an equal increment to unequal distances is, *relative to these distances*, the greater in the proportion as the distances are smaller. If therefore the distance between the resultants of the side-forces be, for instance, three times as great as the distance between the end-surfaces, the distance between the former is increased by only one third, when the distance between the latter, is doubled; and consequently the sum of the attractive terms decreases at a slower rate, than the square of the distance between the end-surfaces increases.

But in general, as the distances increase, the difference decreases between the distances, — relative to each other, — of the points of application of the side, and of the end-magnetism, in consequence of which, the collective attraction at great distances

generally tend to become inversely proportional to the squares of the distances between the end-surfaces. In reality they do become, at very great distances, inversely proportional to the squares of the distances between the centres of gravity of the magnetism, distributed on the sides and end-surfaces, or to thesquares of the distances between the poles.

But at these great distances, the effect of the two remote poles becomes more and more marked, when compared with the nearest poles, and increases the rate at which the attraction decreases, in comparison with the increase of distance. The attraction, which during the gradual removal of the magnets from each other, for a long time; only slowly approached to become inversely proportional to the squares of the distances, becomes at greater distances comparatively quick, first inversely proportional to the Cubes, and further on, approaches to become inversely proportional with the Fourth power of the distances between the end-surfaces. This corresponds to the well known fact, that the attraction of magnetic bars, at great distances, approaches asymtotically to become inversely proportional, with the Fourth powers of the distances between the middles of the bars.

The magnetic forces may in general, at great distances, without perceptible error, be considered as located in the poles, and the attraction, as remarked, to be equal to the sum of the products of the quantities of magnetism, divided by the squares on the distances of the respective poles. In reality the points of application are somewhat nearer to the opposite magnet, than the poles themselves, and a calculation in which the points of application are supposed to coincide with the poles, will therefore give somewhat too small values for the attraction, and this will of course be the case in a higher degree, in proportion as the distances between the magnets are smaller.

We will for a moment consider this relation in the event of the distances between the end-surfaces being 300, 350, 400, 450 and 500 millimeters.

According to the method mentioned at page 72, the positions of the poles in the free magnets are found to be: (See Pl. VI).

In the magnet A: — The negative pole — to the left — is 21.9^{mm}, and the positive 19.55^{mm} from the respective nearest end-surfaces; In B: the negative pole is 19.11^{mm} and the positive 23.18^{mm} from the end-surfaces. The free magnetism imagined to be concentrated in A's poles is 13.7 and in B's 12.3.

The effect consequently, when the distance is: —

$$300^{mm}, \text{ is } \frac{13.7 \cdot 13.2}{(300 + 19.11 + 19.55)^2} - \frac{13.7 \cdot 13.2}{(300 + 19.55 + 122.82)^2}$$
$$- \frac{13.7 \cdot 13.2}{(300 + 19.11 + 124.10)^2} + \frac{13.7 \cdot 13.2}{(300 + 122.82 + 124.10)^2}$$
$$= 0.000338$$

when 350^{mm}	0.000218
„ 400^{mm}	0.000145
„ 450^{mm}	0.0001005
„ 500^{mm}	0.0000718

The relation between the attractions at 300^{mm} and 350^{mm} is inversely proportional to the distances in the

2.841th power	$= a^{-2.841}$
at 350^{mm} and 400^{mm}	$a^{-3.054}$
„ 400^{mm} „ 450^{mm}	$a^{-3.118}$
„ 450^{mm} „ 500^{mm}	$a^{-3.189}$

By a comparison between these calculated and measured attractions, the following relations are obtained: —

	Calculated	Measured
$\frac{\text{Attraction at distance } 300^{mm}}{\text{Attraction at distance } 350^{mm}}$	1.55	1.67
$\frac{350^{mm}}{400^{mm}}$	1.50	1.57
$\frac{400^{mm}}{450^{mm}}$	1.44	1.52
$\frac{450^{mm}}{500^{mm}}$	1.40	1.48

It will be remembered that it has been previously stated, that the quotients found by calculation, in which poles and points of

application of the forces are supposed to coincide, must be somewhat smaller than those corresponding to the actual attractions.

When a magnet and keeper are brought under the influence of each other, the magnetism induced in the keeper is gradually reduced, as the distance between the components of the couple is increased, and the attraction is therefore diminished at a much greater rate, than when two magnets act on each other. From the tables of attraction, page 119, between magnet and armature of equal dimensions (10.5^{mm} diam.) it will be observed, that the attractions — as in the case when two magnets influence each other, — *at short distances* decrease very slowly, as compared to the rate at which these distances *relative to each other* change, whereas, as the distances become greater, the circumstance is entirely reversed; thus at the distance 0.275^{mm} or about $^1/_{20}$ radius the attraction decreases inversely as the Square Root of the distance;

at 1.7^{mm},	= about	$^1/_3$	radius,	inversely	as the	1st	power	of the dist.
„ 11.5^{mm}	„	$2^1/_5$	„	„	„	2nd	„	„ „
„ 57.0^{mm}	„	$10^1/_3$	„	„	„	3rd	„	„ „
„ 86.0^{mm}	„	$16^1/_3$	„	„	„	4th	„	„ „
„ 110.0^{mm}	„	21	„	„	„	5th	„	„ „

etc. When two magnets, on the contrary, attract each other, the attraction can, as mentioned, never decrease more rapidly than inversely as the fourth power of the distance.

When the magnet and keeper have unequal diameters, and are in contact, or at short distances from each other, the direction in which the molecules in the larger of the two components saturate each other, converges towards the end-surface of the smallest component, as shown page 88, by which most of the magnetism becomes collected opposite this surface. The moments in the keepers are therefore approximately equally great, without regard to the diameter, when this is not very much smaller than the

diameter of the magnet. In the series of observations, page 90 and following, it will be found that the moment, and therefore the total free magnetism is always somewhat less in the smaller, than in the larger keepers. In those of 5.5^{mm} and 10.5^{mm} diam. for example, whose transverse areas are to each other as 0.32 : 1, the moments of the end-surfaces at contact have the ratio 0.87 : 1. (See Plate V). When the components are separated, the reaction becomes weaker, and the magnetism attains more of its natural distribution on the end-surface of the larger magnet, it becomes consequently more diffused over the whole surface, and, as the distance increases, accumulates more towards the periphery. But here it is at a greater distance from the keeper, than at the more central part of the surface, and acts besides at a more unfavourable angle, in consequence of which, the moments in smaller keepers, relative to those in larger, are to some extent smaller, when the keeper is more or less removed, than when at contact. At the distance 0.284^{mm} the above relation (87 : 1) is reduced to 0.73 : 1; but is not reduced much lower at the far greater distance 5.32^{mm}, where it is 0.71 : 1. The moments in keepers are generally greater when the keepers themselves are larger; it is true that this has not been found by actual measurement for distances greater than 5.32^{mm}; but that it holds good for all distances may be concluded from the fact, that the attractions at all large distances are less for the smaller than for the larger armatures; and this is by no means owing to any difference in the moments of the magnet, caused by the different keepers, for not only is the reaction nearly equal from all keepers, but at distances, not even much exceeding the diameter of the magnet used, the reaction of the keeper causes hardly any change whatever in the moments of the magnet.

It seems, that the conclusion to be drawn from what has just been mentioned, must be, that the cause of the differences in the moments of keepers of unequal diameters must mainly be ascribed to the resistance to the molecular torsion, increasing at a quicker rate than the torsion itself. If this be the true explanation, then

the loss of moments in the smaller keepers must be proportional to the diminution of their transverse sections.

In keepers of 10.5^{mm}, 8.0^{mm} and 5.5^{mm} diameter, the areas have to each other the ratios 3.65 : 2.12 : 1.00; the moments, when the keeper is in contact with the magnet, have at the contact-surfaces the ratios 9.5 : 9.0 : 8.3. The difference between the greatest and smallest moment is therefore 1.2. If the above conclusion be correct, then the difference between the moments of the 8.0^{mm} and 5.5^{mm} keeper at contact, should

be $= \frac{2.12 \cdot 1.20}{3.65} = 0.70$. The real measured difference (9.0 — 8.3), is also 0.70.

It is not possible by calculation from the moments rigidly to deduce the corresponding attractions between a magnet and keepers of unequal diameters at varying distances, and by means of actual measurements of the attractions to prove or disprove the above conclusion, as to the resistance to the molecular torsion increasing at a more rapid rate than the torsion itself; for sundry relations would have to be taken into account, the nature of which is not known; such as the threefold relation that exists between attraction, distance and diameter of surfaces, and of the mode in which the distribution of the moments on the end-surfaces changes at varying distances, when a magnet and keeper of unequal diameter act upon each other. But so far as the conclusions go, that may be drawn from the supposition named, they certainly agree with the facts, and corroborate the hypothesis. I will specially point out one such fact, with regard to the attractions at great distances, which would be a necessary consequence of the supposition in question, if correct. According to it, as the magnitude of torsion in general becomes less, it should become more nearly proportional to the deflecting power; and from this it follows, that as the distances between magnet and keeper increase, so the attraction should tend to become equally great, for keepers of unequal diameters. This is also really the case, for as the distances increase, the difference between the attractions exerted by

keepers of various diameters becomes always less, except in the event of the components being comparatively near each other, when another factor exerts an effect, which wholly obliterates the one of which we have just spoken. Though the matter last alluded to has been considered before at page 89, yet it may be well to add a few more observations.

The moments, and therefore the free magnetism, in a keeper whose diameter is equal to that of the magnet, are always larger, than in keepers of smaller, and less, than in keepers of larger diameters; the attraction between the magnetic couple in general is naturally stronger or weaker, according as the free magnetism is greater or less. Nevertheless it happens, that when the distances are small, a relation exists, quite opposite to that named, in fact, the attraction at a given distance is then *greater*, when up to a certain degree the diameter of the keeper is smaller than that of the magnet, and vice versa *less*, when the diameter of the keeper is larger, or in other words, the attraction of the magnet towards the magnetically weaker keeper is greater than towards the stronger.

From the tables of attraction it will be seen, that the keeper of 13.0^{mm} diam. reaches its greatest attraction, compared with that of the keeper of 10.5^{mm} diam., at the distance of about 8^{mm}, or $1^1/_2$ rad of the magnet, where the relation between the attractions is as 1.16 : 1.00; and that the keepers of 8.0^{mm} and 5.5^{mm} diameter attain their least attraction, compared with that of 10.5^{mm}, at the respective distances of about 1 and $^3/_5$ of the magnets radius (5.4^{mm} and 3^{mm}) respectively, where the relations are as 1.00 : 0.87 and as 1.00 : 0.68. As the distances become greater, the attractions tend more and more to become equally great for all keepers, without actually becoming so, as previously stated. But as the distances decrease from those named, the attractions converge not only to equality, — which they attain at a distance of about $^1/_5$ of the magnets radius (about 1^{mm}) — but from this point till contact, the former relation is reversed, and thus the smallest and magnetically weakest keepers exert the greatest attraction, and the largest and strong-

est keepers the least attractions. At the distance of 0 00095 radius of magnet (0.05^{mm}) the attraction of the keeper 10.5^{mm} diam. is to that of the largest as 1 : 0.884, and to the two smaller ones as 1 : 1.11 and as 1 : 1.37 respectively.

The explanation of this fact, apparently so paradoxical, is given at page 89.

In Electro-Magnets under the influence of an armature, forces are brought into play, partly different from those in operation in permanent magnets; the laws which regulate the attractions, and consequently the rate, at which the attraction varies with the distance between the attracting components, is therefore entirely different in electro-magnets from those of permanent magnets, whenever these distances are within such limits, that the armature is capable of exercising any perceptible influence on the molecular condition of the magnet.

It has previously been shown, that in free permanent magnets the axial deflection of the molecules is less, the nearer they are to the ends of the bar, and the reason of this is, that the molecules at the end-surfaces, in their exertion to saturate each other, are deflected to a position, in which the direction of their axes is more parallel to the end-planes, than in the position given to them by the magnetization. But as the coercive force, or the friction between the molecules, always opposes their sliding one against the other, the deflection can in general only take place in consequence of the elasticity, that exists in the molecules themselves or in their connections, which allows the axes to take up a modified angular position, within certain limits, while the molecules themselves do not thereby alter their relative positions otherwise. A tension is however produced by this, which, whenever saturation of the molecules can be effected in such direction, forces the axes back to that angular direction, naturally imparted to them by the magnetization. But by such angular recession the stress is continually diminished, and ceases althogether, if the molecules,

— by the magnet being brought in contact with another magnet of equal dimensions, and with the same amount of free magnetism or equal molecular deflection at the contact-surface, — attain the position corresponding to their degree of magnetization, in which, at the same time, they saturate each other completely at the surface of contact. The molecules have then no tendeney to any further alteration of their angular positions. If a stronger magnet be applied to the end, it will certainly cause a greater deflection, but it will be attended by an opposition, which the molecules exert against deflection, beyond the above named angle, and thus a stress in a reverse direction, striving to diminish the deflection, will be the consequence.

In the electro-magnet the case is very different. It is here the exterior electric current, which deflects the molecular axes in the direction of the axis of the iron-bar, in the same manner, as the coercive force does in the free magnetized steel-bar. But the force of the current is not diminished by the deflection, which in the steel bar weakens the magnetic stress.

When the current is opened, the molecules are deflected in the iron bar surrounded by the current, until the resistance against deflection, principally emanating from the free ends of the bar, or from the armature, when joined, is brought into equilibrium with the deflecting force.

When the free end of the electro-magnet is approached by an armature, the latter saturates to a greater or less degree the free molecular poles at the end, whereby the resistance offered to the deflection that the current strives to impart, is diminished with a force, corresponding to the saturation, and the molecules are thus further deflected; an increased deflection of the molecules of the magnet produces however, a corresponding increase of the deflection of the molecules of the keeper, which therefore further neutralises the opposition to deflection in the magnet, and so on, until the resistance in the keeper, and in that portion of the magnet, which is uncovered by the coil, puts an end to any further molecular torsion.

It is clear that under such circumstances the molecular deflection, and consequently the attraction, becomes far greater and increases much more rapidly in proportion to the diminution of the distances, than when, as in the permanent magnet, the deflection tends towards a definite limit, beyond which it can not pass, even if the resistance to the molecular deflection in the keeper offered no hindrance.

For the measurements undertaken to determine the attraction of electro-magnets, the current was so regulated that, at such distances, that the reaction from the keeper exerted no perceptible influence on the molecular condition of the magnet, the attraction of the electro-magnet was made precisely equal to that of the permanent magnet A. At these great distances, the relation between attraction and distance is the same throughout for both kinds of magnets; but as soon as the armature is brought near enough to exert any influence on the magnitude of the moments, the attraction of the two magnets begins to differ, and the difference increases rapidly with the diminution of the distance.

From the table of attractions, page 124, it will be seen, that when the keeper is brought near the magnet, from a greater distance, the attraction is equal for both kinds of magnetism, till, at the distance of about $3^1/_2$ rad. (18^{mm}) between the components, it begins in the electro-magnet to increase at a rate, always more and more rapid, compared to that in the permanent magnet, so that at the distance of 0.00095 rad. (0.05^{mm}) it corresponds in the former to 300 grams, but in the latter to only 113.2 gr.; the relation between the two attractions being then as $2^2/_3 : 1$.

IV.

Before proceeding to describe the mode adopted and the results obtained, by the measurements which have been performed to ascertain the attractive force of magnets and armatures of different kinds, at distances varying within the greatest and smallest limits for which reliable results, by the means employed, could be determined, I shall beg to make a few preliminary remarks.

The two permanent magnets used were the same as those, which before have been referred to, on various occasions, and named A and B. The free magnetism in both was equal at the two ends which were brought opposite each other, but the total amount of magnetism in A was somewhat greater than that in B. For the measurements of attraction of a magnet opposite keepers of different diameters, A was employed. As likewise stated before, the strength of the electro-magnet was regulated so, that at distances beyond 18^{mm} the force exerted by it, was precisely equal to the force of A at these distances.

The measurements have been performed with all the care, I have been able to bestow upon them, and the results obtained, though of course not free from errors, do, I am convinced, generally, within very small deviations, express the true relation between the attractions and distances; yet at distances so great, that the attractions amounted to only a few milligrams, it is almost needless to say, that the difficulties of exact measurements have been so great, that the same scale of precision cannot apply to these, as for the measurements in general.

For determining the force of attraction at short distances, the magnet was moved away from the keeper, till equilibrium was obtained between the attraction and a certain weight, applied to oppose it. In measuring the smallest forces, a balance was employed, in which the magnet was fixed, which therefore had also to be moved. Though this balance was constructed as lightly as was practically possible, it weighed, including its counterpoises, about 1350 grams, and the magnet itself 105 grams. The collected mass to be moved, weighed therefore, about one and a half million times that weight, which corresponds to the smallest attractive forces measured.

It is superfluous to remark that as regards the *individual* observations performed under such exceedingly unfavourable circumstances, there can be no question of accuracy; but by employing the usual means for obtaining reliable results from imperfect observations, I have no doubt, that the determinations made, even for the greatest distances, do within limits of errors but slight express the true attractions. For distances so great, that the parallel series of observations performed showed a want of correspondence, a greater number of observations were made, the mean value of which were laid down as ordinates to abscissæ representing the distances, and then a regular curve was drawn through these points, from which the numbers representing the attractions were read off.

But though the tables no doubt fairly express the true relations existing between the attractions and distances in the magnets and keepers employed, these relations are by no means to be considered as strictly valid for all other magnets, even when they have the same dimensions as those employed for my observations.

It must be remembered that the point of indifference of the moments in magnets does not, at least as a rule, quite coincide with the centre of gravity of the bar, but is generally situated more or less to the side; consequently, the relation between the free magnetism is somewhat different in both sides, and this, in a manner varying, at least to some slight degree, in all magnets.

The relation between the side and end-magnetism of the magnet is not constant, but varies with the relation between the length and diameter of the bar. According to the degree of magnetisation, the extent of the permanent re-deflection of the molecules at the end of the bar is also different, as shown by tables IV, pages 24 and 25, and Plate I, Fig. 1 and 2.

The effect of these and several other circumstances is, that the attractions and distances cannot be in an absolutely fixed relation to each other, even in magnets and keepers having the same dimensions; the relation may even vary in the same pair, according to the degree of magnetisation, which the magnet may have from time to time. But deviations from the causes now mentioned are of course only very slight.

There is however one circumstance, having a far greater importance at comparatively short distances, which it is necessary more fully to consider.

It has been shown (page 68) that in homogeneous magnets of *different* intensity, the magnetic distribution along the axes is similar, or very nearly so, and also, that in keepers of the same shape, the magnetism induced is mainly distributed in equal proportion in the direction of the axes, or that the *relation* between the moments along the axis is not affected by the magnetic intensity, when the distance separating the components is not changed. From this might be concluded, that the attraction of magnets of unequal intensity towards the same keeper, at equal distances, should be proportional to these intensities. Such is also the case, when the distances are comparatively large. But at shorter distances, the attraction increases more rapidly as the distances are diminished, in stronger than in weaker magnetic couples, when one or both components of the magnetic couple consist of soft iron. This will appear from the annexed table, in which the radius of magnet and keeper is 5.25^{mm}; the distances are expressed in $^1/_{1000}$ radius, and the unit of distance consequently $= 0.00525^{mm}$. In the columns I and i those attractions are given in grams.

The magnet appertaining to the series I is the same as one of the equally strong magnets, used in the series III, and the measurements in both series were performed in immediate succession, so that the intensity of the magnet may be considered to have been equal for both.

	I			II			III		
Distance	Perman. magnet and keeper			Electro-magn. and keeper			Two perm. magn. of equal intensity		
	I	i	$\frac{I}{i}$	I	i	$\frac{I}{i}$	I	i	$\frac{I}{i}$
15	103.0	62.5	1.65						
25	88.5	55.0	1.61	164.0	44.0	3.73	149.0	110	1.35
50	67.0	43.0	1.56	99.0	27.5	3.60	116.0	91	1.27
75	54.0	34.0	1.59	69.0	19.5	3.54	96.0	76	1.26
100	44.5	28.2	1.58	53.0	14.5	3.58	83.0	65	1.28
150	32.3	20.5	1.57	32.2	9.50	3.39	62.0	51	1.27
200	24.6	16.0	1.54	23.0	6.85	3.54	52.0	41	1.27
250	19.4	12.6	1.54	17.1	5.00	3.42	44.0	35	1.26
300	15.9	10.5	1.52	13.2	4.00	3.30	37.0	30	1.23
350	13.1	8.9	1.47	11.0	3.20	3.44	32.0	26	1.23
400	11.3	7.5	1.51	8.7	2.55	3.41	29.0	23	1.28
450	10.0	6.5	1.54	7.3	2.20	3.32	26.0	20	1.30
500	8.65	5.65	1.53	6.3	1.92	3.28	22.8	18	1.27
600	6.80	4.42	1.54	4.9	1.50	3.27	19.0	15	1.27
700	5.43	3.62	1.50	3.75	1.22	3.07	16.1	12.7	1.27
800	4.46	3.05	1,46	3.02	1.02	2.96	13.8	11.0	1.25
900	3.80	2.60	1.46	2.52	0.87	2.90	12.2	9.5	1.28
1000	3.26	2.21	1.47	2.10	0.74	2.84	10.7	8.5	1.26
1200	2.46	1.74	1.42	1.57	0.56	2.80	8.7	6.7	1.30
1500	1.76	1.20	1.47	1.20	0.43	2.79	6.6	5.3	1.25

Although there are small discrepancies in the values of the above quotients, still it clearly follows, that those belonging to the series I are on the whole constant between the distances 1500 and 800 (1.5 and 0.8 rad.), but that from the last mentioned point the values in the column I are, as compared with those in column *i*, gradually increasing inversely with the distances, whereas such is at least the case only to a very slight degree in the series III. But in II, the increase of I over *i* is very great, being appreciable even between the distances 1500 and 800.

As the relation between the distribution of the moments *along the axes* at equal distances between magnet and keeper is not affected by the intensity of the magnetism, it seems beyond doubt, that the variation between the relative attractions pointed out must be dependent on inequality in the relative distribution of the moments in the planes, *transverse to the axis,* mainly of course on the end-surfaces; — which, as previously shown, will produce an effect similar to the one here spoken of.

On account of the greater accumulation of magnetism towards the periphery, the effect of the magnet on the armature is transmitted somewhat differently, when the distance between the components is small, to what it is when large.

It is clear that the effect upon the periphery of the end-surface of the keeper, of the magnetism accumulated in the periphery of the magnet, must, at great distances, be nearly uniform on all points, as well in the periphery as near the centre. But at short distances the effect is more local, and consequently greater on and near the opposite points of the periphery, than on any other part. At contact the whole inductive effect of the magnet's peripheric force is in reality transmitted to the keeper through its periphery.

It has been mentioned at pages 78 and 79, that even when the inductive force is transmitted to, and distributed from, the centre of one end of the keeper, the molecules attain a deviation, which at the other end near the centre is small, but towards the periphery increases at a rapid rate. Much more must this be the case, when the points of application of the inductive force are those of the periphery itself, or parts of the surface near it, and the induction therefore is distributed thence towards the centre..

The conditions on which the equilibrium between moments of varying magnitude depends, such as they are distributed over the end-surface, or any transverse section of a magnetic bar, — not being known, it cannot be concluded *a priori,* whether or not an altered intensity of the magnet will cause any difference in the *relation* between the moments induced on the transverse sur-

face of the armature. It clearly follows from the table, that at large distances between the components, no change in this relation is caused, as the quotients $\frac{I}{i}$ remain constant; but in proportion as the distances become shorter, and the induction of the armature emanates principally from the periphery, it seems highly probable, that when the intensity of the induction is increased, the moments become *comparatively* greater near the periphery of the armature, than at the centre, and thus a greater concentration of the free magnetism takes place in a narrower belt at the periphery, which, as shown in Table C, page 86, and page 89, would cause an increase of the attractive power, although the quantity of the free magnetism remains unaltered.

That it is in the soft iron armature alone or principally, that the modification of the moments, relatively to each other, takes place, seems to follow from the fact, that when two magnets attract each other, the quotients remain on the whole unaltered, and that in the case of an electro-magnet and keeper, where both the attracting bodies consist of soft iron, the effect is greatest.

With regard to electro-magnets, a modification in the rate at which the attraction varies with the distance from the armature, is also produced by the position of the coil upon the magnetic bar, in so far, as it does not cover the whole of this bar, and by the length of the coil, as compared with the length of the bar.

Measured Attractions.

Attraction between Magnet 146mm in length, 10.5mm diameter and Armatures of same length, 13.0mm, 10.5mm, 8.0mm and 5.5mm diam.

Distance in Millim.	Attraction in Grams 13.0mm	10.5mm	8.0mm	5.5mm
0.050	100.1	113.2	125.1	155.3
0.075	92.5	104.5	114.0	139.8
0.100	86.0	96.9	104.4	125.8
0.125	80.5	90.4	96.2	113.1
0.150	76.0	84.9	89.2	101.9
0.175	72.0	80.2	83.2	91.8
0.200	68.5	76.1	78.2	82.9
0.225	65.4	72.4	73.8	75.1
0.250	62.6	69.0	69.9	68.3
0.275	60.0	65.9	66.3	62.2
0.300	57.5	63.0	62.9	57.0
0.325	55.2	60.3	59.8	52.8
0.350	53.0	57.7	56.8	49.3
0.375	50.9	55.2	54.0	46.3
0.400	48.8	52.8	51.4	43.5
0.450	45.2	48.6	46.9	39.5
0.500	42.1	45.0	43.1	36.2
0.550	39.4	41.9	39.9	33.2
0.600	37.2	39.4	37.3	30.7
0.700	33.5	35.1	32.8	26.6
0.800	30.4	31.5	29.0	23.4
0.900	27.4	28.2	25.8	20.6
1.000	24.7	25.3	23.0	18.2
1.100	22.6	22.9	20.7	16.3
1.200	20.8	20.9	18.8	14.7
1.300	19.2	19.2	17.2	13.4
1.400	17.8	17.7	15.8	12.3

Distance in Millim.	Attraction in Grams 13.0mm	10.5mm	8.0mm	5.5mm
1.5	16.6	16.4	14.6	11.4
1.6	15.7	15.4	13.6	10.7
1.7	14.8	14.5	12.8	10.0
1.8	14.0	13.6	12.0	9.40
1.9	13.3	12.8	11.3	8.83
2.0	12.6	12.1	10.6	8.30
2.2	11.3	10.8	9.40	7.34
2.4	10.2	9.60	8.36	6.54
2.6	9.20	8.57	7.46	5.86
2.8	8.29	7.74	6.73	5.29
3.0	7.58	7.04	6.10	4.77
3.2	6.96	6.43	5.56	4.34
3.4	6.43	5.91	5.11	4.00
3.6	5.97	5.45	4.73	3.72
3.8	5.58	5.05	4.39	3.45
4.0	5.23	4.71	4.09	3.21
4.2	4.91	4.41	3.83	3.00
4.4	4.62	4.14	3.59	2.82
4.6	4,34	3.89	3.37	2.65
4.8	4.09	3.65	3.16	2.49
5.0	3.86	3.42	2.96	2.33
5,2	3.65	3.22	2.79	2.19
5.4	3.45	3.04	2.63	2.06
5.6	3.26	2.88	2.48	1.95
5.8	3.09	2.72	2.34	1.85
6.0	2.93	2.56	2.22	1.74
6.5	2.56	2.24	1.94	1.53
7.0	2.24	1.95	1.69	1.33
7.5	1.97	1.71	1.48	1.17
8.0	1.76	1.51	1.31	1.04
8.5	1.58	1.36	1.18	0.930
9.0	1.43	1.23	1.07	0.840
9.5	1.29	1.11	0.966	0.764

Distance in Millim.	Attraction in Grams 13.0mm	10.5mm	8.0mm	5.5mm
10.0	1.17	1.01	0.879	0.696
10.5	1.07	0.923	804	637
11.0	0.981	844	736	583
11.5	898	773	674	535
12.0	823	709	619	491
12.5	755	651	569	452
13.0	695	600	525	417
13.5	640	553	484	385
14.0	590	510	447	355
14.5	547	473	414	330
15.0	506	438	384	306
15.5	469	406	357	284
16.0	436	378	332	265
16.5	405	351	308	247
17.0	376	326	286	230
17.5	349	303	266	214
18.0	325	282	248	200
18.5	304	264	233	187
19.0	285	248	218	175
19.5	269	234	206	165
20.0	255	222	196	157
21.0	229	200	177	142
22.0	206	180	159	128
23.0	185	162	143	115
24.0	165	145	128	104
25.0	148	130	115	936
26.0	133	118	104	842
27.0	120	106	942	767
28.0	109	965	858	700
29.0	989	879	782	640
30.0	899	801	714	585
31.0	818	730	652	534
32.0	745	666	595	489

Distance in Millim.	Attraction in Grams 13.0mm	10.5mm	8.0mm	5.5mm
33	0.0681	0.0610	0.0545	0.0449
34	623	560	501	413
35	573	516	462	382
36	528	476	426	353
37	489	441	395	328
38	454	410	368	305
39	422	382	342	285
40	394	357	321	268
41	369	335	302	252
42	346	315	284	237
43	325	296	267	223
44	305	278	251	210
45	287	262	236	198
46	270	247	223	187
47	254	233	211	177
48	239	220	199	168
49	226	208	188	159
50	214	197	178	151
51	203	187	169	143
52	193	178	161	136
53	183	169	153	130
54	174	160	146	124
55	165	152	139	118
56	156	144	132	112
57	148	137	125	106
58	140	130	118	101
59	132	123	112	957
60	125	116	106	904
62	112	104	949	813
64	101	945	863	740
66	914	855	781	672
68	824	773	707	609
70	743	698	639	552

Distance in Millim.	Attraction in Grams 13.0mm	10.5mm	8.0mm	5.5mm
72	0.00669	0.00629	0.00576	0.00499
74	603	568	521	451
76	544	513	471	408
78	491	464	426	369
80	443	419	385	334
82	401	380	349	304
84	365	346	318	277
86	333	315	290	252
88	302	287	264	230
90	275	262	241	210
92	251	239	219	192
94	228	217	200	175
96	208	198	183	160
98	190	181	167	146
100	173	165	153	133
102	158	151	140	122
104	145	139	129	113
106	133	128	119	104
108	122	117	109	95
110	111	107	99	87
112	102	98	91	80
114	93	89	83	73
116	85	82	76	67
118	78	75	69	61
120	71	69	64	56

Attraction between Electro-Magnet and Armature, both 146^{mm} long and 10.5^{mm} diameter.

Distance	Attraction	Distance	Attraction
0.050^{mm}	300.0^{gr}	1.8^{mm}	16.4^{gr}
0.075	256.2	1.9	15.4
0.100	224.5	2.0	14.6
0.125	200.9	2.2	12.8
0.150	181.9	2.4	11.3
0.175	166.3	2.6	10.0
0.200	153.5	2.8	8.93
0.225	142.9	3.0	8.06
0.250	133.4	3.2	7.32
0.275	124.8	3.4	6.68
0.300	117.0	3.6	6.12
0.325	109.9	3.8	5.63
0.350	103.3	4.0	5.20
0.375	97.0	4.2	4.83
0.400	90.9	4.4	4.51
0.450	80.9	4.6	4.23
0.500	72.8	4.8	3.98
0.550	66.2	5.0	3.75
0.600	60.8	5.2	3.53
0.700	51.9	5.4	3.32
0.800	45.0	5.6	3.12
0.900	39.5	5.8	2.93
1.000	35.0	6.0	2.75
1.100	31.2	6.5	2.40
1.200	27.9	7.0	2.10
1.300	25.1	7.5	1.84
1.400	22.7	8.0	1.62
1.500	20.7	8.5	1.43
1.600	19.0	9.0	1.27
1.700	17.6	9.5	1.14

Distance	Attraction	Distance	Attraction
10.0^{mm}	1.03^{gr}	16.0^{mm}	382^{gr}
10.5	0.940	16.5	353
11.0	862	17.0	327
11.5	794	17.5	304
12.0	732	18.0	283
12.5	675	18.5	264
13.0	622	19.0	248
13.5	573	19.5	234
14.0	528	20.0	222
14.5	487	21.0	200
15.0	449	22.0	180
15.5	414	23.0	162

At greater distances the attractions are as for permanent magnet and armature.

Attraction between two Permanent Magnets of equal intensity 146^{mm} long and 10.5^{mm} diam.

Distance	Attraction	Distance	Attraction
0.050^{mm}	184.4^{gr}	0.55^{mm}	78.6^{gr}
0.075	173.2	0.60	75.0
0.100	163.1	0.70	68.5
0.125	153.5	0.80	62.9
0.150	144.2	0.90	58.1
0.175	136.0	1.00	53.9
0.200	128.8	1.10	50.2
0.225	122.5	1.20	46.9
0.250	117.0	1.30	43.9
0.275	112.2	1.40	41.2
0.300	108.0	1.50	38.8
0.325	104.1	1.60	36.6
0.350	100.4	1.70	34.6
0.375	96.9	1.80	32.7
0.400	93.6	1.90	31.0
0.450	87.8	2.00	29.5
0.500	82.8	2.20	26.8

Distance	Attraction	Distance	Attraction
2.4mm	24.5gr	14.0mm	3.12gr
2.6	22.5	14.5	2.98
2.8	20.8	15.0	2.84
3.0	19.4	15.5	2.71
3.2	18.1	16.0	2.58
3.4	17.0	16.5	2.46
3.6	16.0	17.0	2.35
3.8	15.1	17.5	2.25
4.0	14.3	18.0	2.16
4.2	13.6	18.5	2.07
4.4	13.0	19.0	1.99
4.6	12.4	19.5	1.92
4.8	11.8	20.0	1.85
5.0	11.3	21.0	1.72
5.2	10.8	22.0	1.61
5.4	10.3	23.0	1.51
5.6	9.84	24.0	1.42
5.8	9.41	25.0	1.34
6.0	8.99	26.0	1.26
6.5	8.12	27.0	1.19
7.0	7.45	28.0	1.12
7.5	6.89	29.0	1.06
8.0	6.38	30.0	1.00
8.5	5.93	31.0	0.950
9.0	5.52	32.0	904
9.5	5.15	33.0	862
10.0	4.81	34.0	822
10.5	4.51	35.0	785
11.0	4.24	36.0	751
11.5	4.00	37.0	719
12.0	3.79	38.0	689
12.5	3.60	39.0	661
13.0	3.43	40.0	634
13.5	3.27	41.0	608

Distance	Attraction	Distance	Attraction
42mm	0.583gr	82mm	0.174gr
43	560	84	166
44	539	86	158
45	519	88	151
46	500	90	144
47	482	92	138
48	465	94	132
49	449	96	126
50	433	98	121
51	418	100	116
52	404	102	111
53	391	104	106
54	378	106	102
55	366	108	979
56	655	110	941
57	344	112	907
58	334	114	875
59	324	116	844
60	314	118	815
61	305	120	787
62	296	122	759
63	287	124	732
64	279	126	706
65	271	128	681
66	264	130	657
67	257	132	634
68	250	134	612
69	243	136	592
70	236	138	572
72	224	140	552
74	213	142	534
76	202	144	517
78	192	146	501
80	183	148	485

Distance	Attraction	Distance	Attraction
150^{mm}	0.0468^{gr}	240^{mm}	0.0130^{gr}
152	452	245	121
154	438	250	113
156	424	255	106
158	411	260	997
160	398	265	939
162	386	270	885
164	374	275	833
166	362	280	783
168	351	285	737
170	341	290	694
172	331	295	655
174	321	300	619
176	311	310	553
178	302	320	496
180	294	330	447
182	285	340	405
184	277	350	369
186	270	360	337
188	263	370	308
190	256	380	281
192	248	390	257
194	241	400	235
196	234	410	215
198	227	420	197
200	221	430	181
205	206	440	167
210	193	450	154
215	180	460	142
220	168	470	131
225	158	480	121
230	148	490	112
235	139	500	104

The foregoing measurements were performed by means of two apparatus, the construction of which will be briefly described.

On a horizontal foot-plate of wood are erected two slender brass columns, *a a*, in the upper ends of which a screw, *b b*, can be turned. At the end of each screw is a groove, in which a minute ring is laid, formed at the upper end of thin brass-wire, to the lower end of which the keeper *A* is suspended. The wires can be shortened or lengthened by means of screw-couplings, *c c*, and a proper adjustment of the armature in the vertical plane thereby effected, while the adjustment in the horizontal plane is made by the screws *b b*.

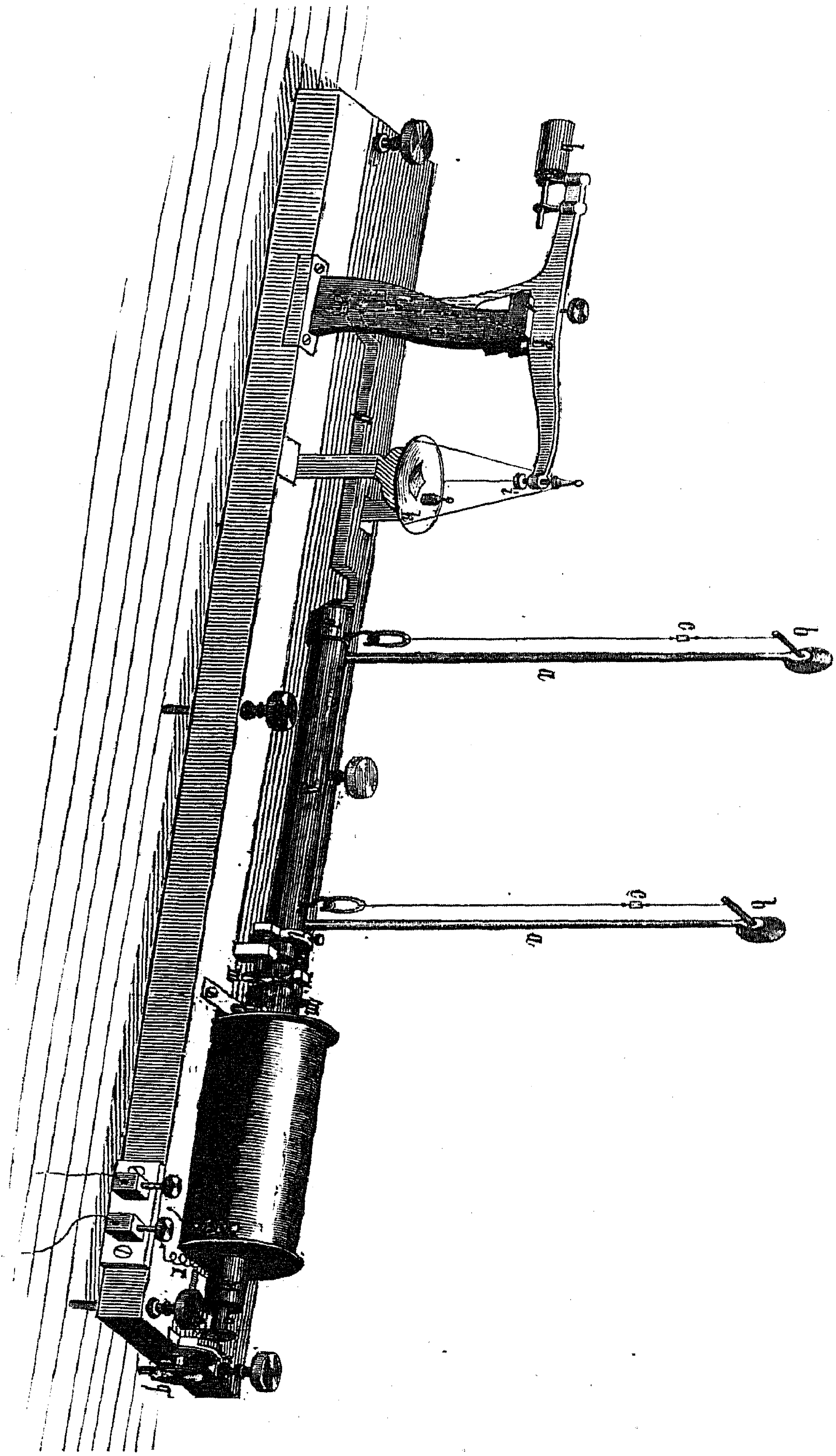
On the left end of the armature is fitted a cap of brass *d*, the interior surface of which at the closed end has the form of a hollow cone of about 45°, terminating in a point. Further towards the left is a stand, in form of a bracket, *e*, on the top of which are two grooves, in which the knives of the balance, *f*, move. This balance has a vertical arm *f'*, whose lower end has the form of a fork, immediately above which on the left hand side a shallow hole is sunk, ending in a hollow conic point. Between this and the hollow conical bearing in the brass-cap of the armature is inserted a connecting-piece of thin brass, *g*, whose hook-formed pointed ends rest against the bottoms of the two conical bearings.

The balance is regulated by the counterpoise *h*, in such manner, that it is only just enabled to press the end-points of the connecting piece, *g*, into the ends of their bearings in *d* and *f'*.

By means of the vertical screw *i*, from whose pointed end the scale, *k*, is suspended, the centre of gravity of the balance is regulated.

On the right hand end of the armature is fixed a collar *l*, with two projections, against which the ends of two horizontal screws *m m*, can be moved.

Opposite the armature is placed the magnet or electro-magnet *M*, with its front-end between two vertical cylindrical guides, *n*, moveable on eccentric pivots in the bed-plate, for the horizontal



adjustment of the magnet. In the opposite end of the magnet is fixed a brass-cylinder, whose free end has the form of a button. This brass cylinder, *o*, rests in a horizontal groove in a vertical projection, *p*, of a slide below, which slide, by means of the micrometer-screw *q*, can be moved parallel to the common axis of the magnet and armature, forwards or backwards, in brass guides, sunk in the foot-plate.

When the balance and armature are joined by means of the connecting-piece *g*, and adjusted as mentioned, the screws, *m m*, are turned forward till they touch *l*, thus preventing the armature from being moved to the right of its present position. The magnet is now laid in its place, and by the micrometer-screw, *q*, moved towards the armature, until its end-surface, which is turned and ground at right angles to its axis, touches the end-surfaces of the magnet, which is also similarly treated, to form a perfect plane at right angles to its axis; between the two planes is, however, inserted a leaf of very thin and hard paper, the thickness of which has first been ascertained.

The position of the slide is now read off on the scale, *r*, along which it moves, as well as the position of the graduated drum, *s*, of the micrometer-screw; and a certain weight is placed in the scale, *k*; the micrometer-screw is turned, and the magnet thereby slowly removed from the armature, which by the attraction is held against the screws, *m m*, without being able to follow the magnet. When the distance between the two end-surfaces is reached, at which the position of equilibrium between the pressure from the weight and the attraction of the magnet is exceeded, the armature is pulled away to the left by the pressure of the weight. By now again reading off the position of the slide and drum, the distance is found.

The length of one turning of the micrometer-screw is $0{,}4813^{mm}$, the drum is graduated into twentieth parts of the circumference, which divisions are far enough apart to enable interpolations of fifths to be made with tolerable correctness; and the distances between the end-surfaces of the magnet and keeper can conse-

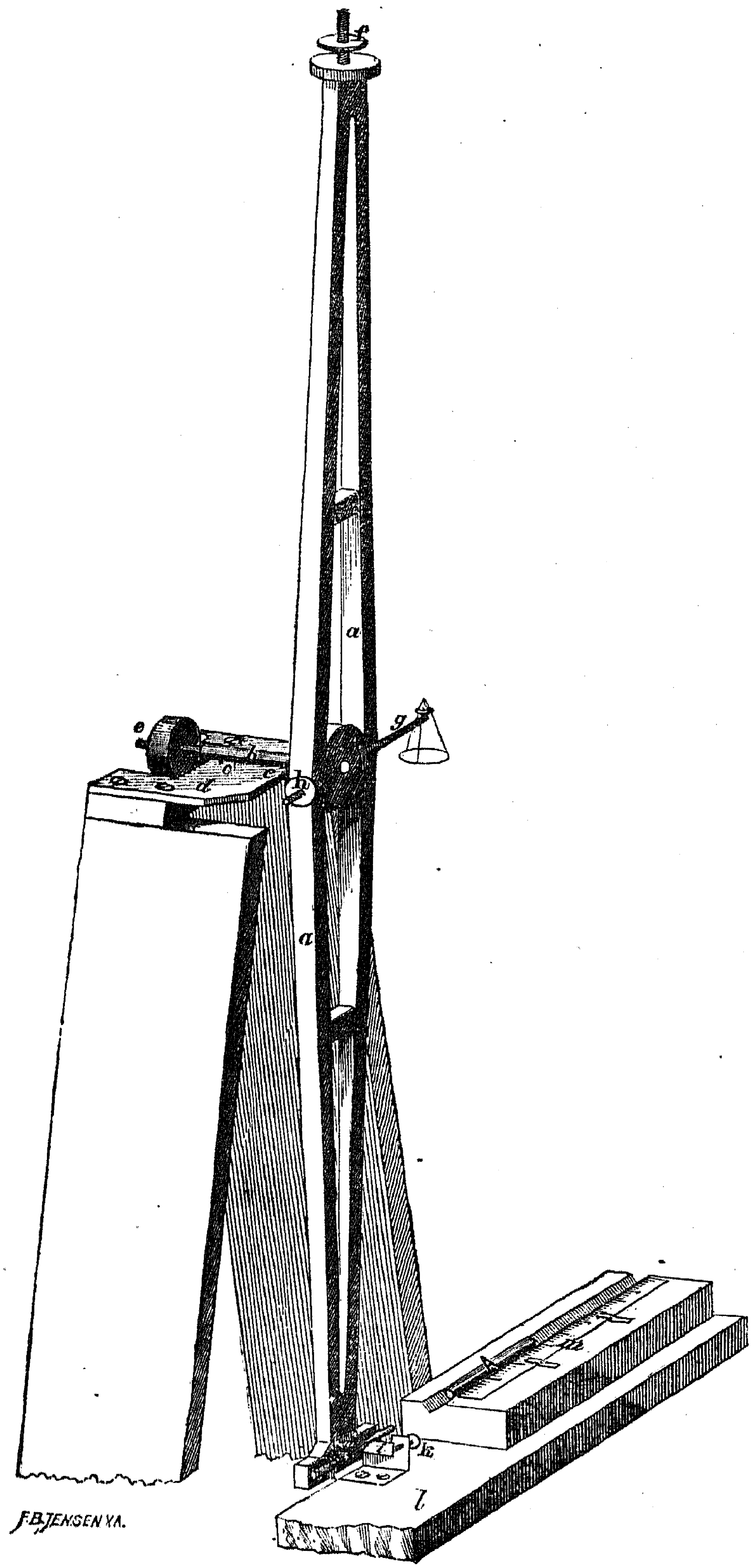
quently be determined within an uncertainty of $0{,}005^{mm}$, provided no regard be taken of the possible errors in the lengths of the individual screw-turnings, which errors, so far as they do exist, are certainly small in the screw employed.

This apparatus has been used for measurements of the attraction, at distances ranging between $0{,}05^{mm}$ and 9.0^{mm}.

For larger distances a vertical balance, *a*, of wood — see next figure — is employed, which, fixed to the axle, *b*, can move upon the nearly needle-pointed end of a vertical screw, *c*. This screw is inserted in a cavity, bored transversely in the axle, — the bottom of which at the centre of the axle is conically pointed at an angle of about 45^0. To prevent the axle swinging horizontally on the screw named, another screw c' is likewise placed in the brass-plate *d*, and inserted in a cavity, of the same depth and shape as the former one. The worm of the latter screw fits tightly into the plate, in the direction transverse to the length of the axle; but the female-worm in the plate is extended parallel to the axle, allowing the end of the screw, to some small extent, to move in this direction. It would, practically speaking, be impossible to execute the whole in such manner, that the pointed ends of the screws should absolutely coincide with the pointed bottoms of the borings in the axle, if both screws were perfectly rigid in all directions, and if such coincidence could not be obtained, it would cause friction, that would highly interfere with the sensibility of the apparatus. By the arrangement named, however, this coincidence is of course effected entirely.

On the other end of the axle, *b*, there is a counterpoise, *e*, the position of which is regulated so that nearly the whole weight of the balance is thrown on the pivot, *c*, while there is however sufficient pressure also on c', to keep the end of the pointed boring down upon the end of the screw.

The magnet, *M*, is fixed at the lower end of the balance, by means of a screw. On the upper end are two counterpoises, of which the smallest and uppermost one, *f*, can be screwed up and


f
a
a
g
e
h
d
c
k
l
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down, whereby the centre of gravity of the balance is regulated to coincide very nearly with its mathematical axis of rotation.

At the centre of the balance is fixed an arm, *g*, from which is suspended the scale for the weights, and on the opposite side, is a lever with a counterpoise, *h*, to be screwed towards or from the axis, for regulating the position of the balance in the vertical plane. When by means of this screw it is regulated so, that the projection, *i*, at the lower end, is very near to, without however touching, the screw *k*, moveable in a bracket fixed to the board, *l*, — and when the balance has been brought to perfect rest, — the screw, *k*, is moved towards *i*, till its pointed end just touches it. By means of this screw the balance is prevented from moving to the right of its present position.

The armature *A* is now laid opposite to the magnet *M*, in a groove, cut in a wooden block, and is moved towards *M*, till the end-surfaces, — within the thickness of a card or strip of paper of known thickness, laid between, — touch each other. In case the axes of the magnet and keeper do not coincide, the position of the magnet is adjusted by means of the screws *c* and *c′*.

After the keeper is drawn so far to the right as the attraction permits, without the use of any pressure to overcome it, i. e. when the bracket *i* rests against the screw *k*, the scale *m* is adjusted so that its zero point coincides with the edge of the remote end-surface of the keeper. The weight to be used is now laid in the scale at *g*, and the keeper slowly withdrawn till *i* leaves the screw *k*; — at that instant the attraction is of course just exceeded by the pressure from the weight.

When the forces are very small, the distance that *i* recedes from *k* is also very slight, even after a readjustment of the counterpoise *f* has been arranged especially for the smallest weights, — and the motion of the balance must therefore be observed by the reflection from the silvered and polished surface of *i*, of the dark point of the screw *k*, — on which the daylight, but not the direct rays of the sun should fall; artificial light,

causing a current of air by the heat of its flame, should be avoided.

As the balance mainly rests and moves upon a single fine point, and consequently experiences very slight friction, the apparatus is highly sensitive. It must however be covered by a hood of paper or cardboard with the least possible openings for the magnet and scale-balance. This is necessary to prevent irregularities from the motion of the surrounding air, which motion, only so far as it is caused by the breath, heat or movements of the observer, is sufficient sensibly to affect the balance on account of its large surface, when the forces measured are very small, and thus produce errors in the accuracy of the results obtained.